Design of a Multi-Channel Radar System

Employing Non-Uniform Arrays and Substrate Integrated Waveguides

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while enrolled in the Master’s degree program Electronics and Information Technology

November 14, 2019
Statutory Declaration

I hereby declare that the thesis submitted is my own unaided work, that I have not used other than the sources indicated, and that all direct and indirect sources are acknowledged as references. This printed thesis is identical with the electronic version submitted.

Signed:

Date:
“All we have to decide is what to do with the time that is given us.”

- Gandalf
JOHANNES KEPLER UNIVERSITY LINZ

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by Simon Hehenberger

Abstract

Radar technology is expected to play a crucial role in future Advanced Driver Assistance Systems (ADAS) and automated driving applications due to its insensitivity to harsh weather conditions as well as its capability to precisely measure range and relative velocity of objects in the road environment. This thesis presents the theory and applied concepts, as well as the design process and verification of a fully functional FMCW MIMO radar sensor intended for automotive applications based on the 77 GHz radar frontend chipset from Infineon. The main goal is to investigate and employ Substrate Integrated Waveguide (SIW) technology for the individual antenna elements and their respective feed structures. Furthermore, the research and implementation of a suitable sidelobe suppression techniques is another main focus.

The resulting system employs an antenna element based on a linear resonant array of narrow longitudinal slots in the upper wall of a SIW. The antennas of the eight transmit, and sixteen receive channels are arranged in a non-uniform planar array delivering 3D spatial positioning capability. A novel structure is introduced to realize a transition from differential microstrip line into a SIW. The behavior of the designed SIW and other individual system parts are verified through scattering parameter measurements utilizing a wafer prober as well as waveguide transitions. An anechoic chamber is employed to measure the antenna beam patterns and to perform single target measurements. Furthermore, the effect of errors in gain and phase for individual channels as well as errors due to mutual coupling effects on measurements are considered, and two different calibration techniques for FMCW MIMO radar systems are employed to compensate these errors. The successful design and manufacturing of the radar system is tested and verified through a link-budget analysis as well as with real-world application scenarios.
Acknowledgements

In front of you lies my master thesis report. It is the final step for me to finish my Master of Science in Electronics and Information Technology at the Johannes Kepler University. This project was carried out partly at Delft University of Technology and is a product of the cooperation between the Microwave Sensing, Signals and Systems (MS3) group in Delft and the Institute for Communications Engineering and RF-Systems (NTHFS) in Linz. This document represents 18 months of diligent work and commitment, and finishing it means a lot to me. At this point, I want to express my gratitude to all who contributed to the successful conclusion of this project.

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Chapter 1

Introduction

In 1886, the German physicist Heinrich Hertz demonstrated the propagation and reflection of Electromagnetic (EM) waves, thus delivering the first experimental verification of the predictions made in 1873 by James C. Maxwell in [1]. The practical value of Hertz’s work was doubted until the year 1901 when Guglielmo Marconi was able to perform the first transatlantic transmission through radio waves, paving the way for wireless telecommunication technology.

'I do not think that the wireless waves I have discovered will have any practical application.' - Heinrich R. Hertz

Shortly after Marconi’s success, Christian Hulsmeyer obtained a British patent for his detection and ranging device, the "Telemobiloscope" [2], which is now considered to be the predecessor of radar systems. The mathematical description and experimental discovery of electromagnetic waves inspired engineers around the world and brought forth applications that changed society as well as the course of wars, and is a significant driver of the global economy up to this day.

1.1 Radar and its Automotive Applications

Topics regarding the detection and ranging utilizing EM waves were eagerly studied during the first half of the 20th century and especially during the second world war, which was also when the United States Navy coined the term Radio Detection and Ranging (RADAR)\(^1\) [3]. During the war, concepts like pulse-Doppler, monopulse, and phased array radars were introduced, and as they matured after the war, paved the way for nonmilitary use of RADAR technology. Air and space traffic control, radio astronomy and meteorological observation are traditional civil applications of radar technology. However, due to the ongoing development of mmWave\(^2\) circuits and the increasing availability of mobile computing power, radar technology found its way into the automotive industry.

In 2010 the European Commission adopted an ambitious road safety program that aimed to cut road deaths in Europe in half during the oncoming decade [4]. One primary strategy to achieve this goal is the utilization of ADAS such as collision

\(^1\)The abbreviation RADAR is sometimes also used to refer to the terms "radio angle detection and ranging" or "radio aircraft detection and ranging".

\(^2\)Commonly used term to refer to devices operating in the spectrum between 30 GHz and 300 GHz.
warning and mitigation systems. Radar-based driver assistance systems are already widely in use for comfort applications such as adaptive cruise control, collision warning systems, blind spot detection, and rear cross-traffic alert [5]. These systems are supposed to form a protective sensing *cocoon* around the car, as suggested in Fig. 1.1. State of the art ADAS employs a variety of sensors such as Light Detection and Ranging (LIDAR), camera, radar, and ultrasound in cooperation with sensor fusion and deep learning algorithms to improve the interaction between the driver and the road environment.

![Diagram](image.png)

**Figure 1.1:** ADAS safety systems implementing a safety *cocoon* around the car, realized by a combination of LIDAR, RADAR and camera sensors. Source: Texas Instruments.

Although camera and LIDAR systems are ideally suited for mapping and understanding the surrounding road environment, these technologies perform poorly on long-range applications as well as in harsh weather and lighting conditions [6]. Radar, on the other hand, is very insensitive to environmental circumstances like visibility issues due to fog, rain or snow and is able to measure distance as well as the relative velocity of individual objects with high accuracy. Furthermore, radar systems are of lower cost, mechanically less bulky, and can be neatly hidden behind bumpers\(^3\) for aesthetic reasons. The continuing push for ADAS and automated driving is currently the most significant force behind the development of low-cost and high-resolution radar transceivers. Semiconductor companies like Analog Devices, Broadcom, NXP, TI, and Infineon, are eager to position themselves as a reliable supplier of highly integrated radar transceiver systems, since the market is expected to increase with an annual growth rate of more than 20 percent at least until 2025 [7].

\(^3\)This is a commonly stated benefit although the effort for integration is significant and additional losses are easily in the range of 6 - 10 dB.
1.2 State of the Art and Current Challenges

In contrast to conventional phased array radar systems, current state of the art automotive radar sensors employ multiple transmit (input) antennas to send orthogonal Frequency Modulated Continuous Wave (FMCW) signals and receive their echo with multiple receive (output) antennas. The employment of such a Multiple Input Multiple Output (MIMO) antenna configuration increases the available degrees of freedom to the engineer. These degrees of freedom can be exploited to improve resolution, clutter mitigation, and classification performance [8]. The FMCW modulation requires only low power for transmission and can thus be supplied by cheap solid-state components, while delivering excellent range resolution and a small form factor of the sensor. However, the range of such systems is limited and FMCW radars are quite susceptible to interference from other automotive radar systems. Current radar sensors for automotive applications deviate all necessary system parts and components into two parts. The first part supports the base-band processing hardware in the form of a digital signal processor or a Field Programmable Gate Array (FPGA) and an interface for communication. The second part, usually called the frontend, contains the antennas and other RF passive components as well as Monolithic Microwave Integrated Circuit (MMIC) for waveform generation, power amplification and receive mixing. These two parts occupy protective housing covered by a radome\(^4\) to protect the sensor from external mechanical and chemical influences. The utilized antennas are most commonly fabricated on the frontend PCB in the form of planar microstrip antennas. Consider for example the Bosch LRR3 sensor [9], which employs four microstrip patch antennas, or the sensor introduced in [10] that uses a chain of microstrip patches connected by half-wavelength lines. Antennas in microstrip form are easy and cheap to manufacture but also show some drawbacks like unwanted radiation from the feed lines, low bandwidth, and low overall efficiency. In recent years another form of planar technology, the so-called Substrate Integrated Waveguide (SIW), was a topic of extensive research and presented itself as a promising alternative. The SIW approximates the behavior of an Rectangular Waveguide (RWG) within the substrate and antennas are manufactured by means of cutouts in the upper waveguide wall. The SIW exhibits higher power handling capabilities and lower radiation loss compared to microstrip technology but is slightly more challenging to fabricate and is more sensitive to manufacturing tolerances. Despite the benefits of SIW, microstrip is still the preferred solution since the transition for signals from the MMIC to the PCB and vice versa is simplest with microstrip technology. However, there is not yet a solution on how to transition from currently employed MMIC packages to SIWs directly. The ongoing integration trend aims to produce a sensor, for which all necessary active and passive components, including the antennas, are integrated within a single chip. However, this goal is years down the road. Current sensor systems still often rely upon individual MMIC for each active task on the frontend, which is great for flexibility but increases cost and the size of the sensor. An excellent example of such a distributed chipset is the current generation of 77 GHz Radar frontend ICs [11].

From an academic perspective, research teams invest extensive effort into antenna array topologies, planar antennas, and efficient planar transmission-line technologies. However, these topics do not concern automotive radar exclusively but concern the RF community as a whole. Current research concerning automotive radar concentrates on

\(^4\)A mechanically stable, weatherproof cover made from a material that minimally attenuates the EM wave as it passes through.
increasing angular resolution and interference cancellation between FMCW systems as well as moving from FMCW to Phase Modulated Continuous Wave (PMCW) systems. Despite significant accomplishments, in the past few years, many issues like height measurement capability, higher spatial resolution, ultra near range detection, and optimal bandwidth usage require further engagement and innovative solutions [12].

1.3 Focus and Structure of this Work

The main goal was to produce a fully functional radar system with the current generation 77 GHz MMICs from Infineon fit for automotive applications. The focus lies on the design of the antenna element, the MIMO array topology and the transition from the individual antenna elements to the integrated circuits. This primary task was approached by splitting it into two work-packages. First, various MIMO array topologies were investigated and, considering design limitations, a suitable topology, and design algorithm with particular attention to sidelobe suppression in the angular domain was selected. In the second work-package, attention was focused on the individual antenna elements and their design by utilizing SIW technology.

This thesis is structured as follows. Chapter 2 introduces the fully functional radar system by explaining the basic requirements upon which it is built, giving an abstract functional description and list the utilized hardware as well as describe the PCB layer-stack. Before shedding light upon the actual design process of the antenna frontend, the necessary basic theory is the topic of Chapter 3. First, an insight into the basics of radar systems in general, and the specialties of FMCW and the MIMO concept is given. Second, the basics of antennas and antenna arrays, as well as a design algorithm for spatial density tapered arrays based on convex optimization, is investigated. The chapter concludes by introducing the SIW technology, comparing it to other waveguides and printed planar transmission lines and explaining how slots can be utilized to introduce radiating elements into the SIW. Chapter 4 explains the details of the design process, starting with the calculation of the SIW dimensions and the design of the antenna element. Further, the topology of the array is found by applying the convex algorithm introduced in Chapter 3, and the transition from SIW to the utilized integrated circuits is investigated. Chapter 5 describes the experimental verification of individual components and presents the results compared to full-wave simulations. Furthermore, tests and measurements of the complete radar system carried out in an anechoic chamber as well as outside in a parking lot are presented for various measurement scenarios.
Chapter 2

Overview

This chapter provides a comprehensible overview of the completed and fully functional FMCW MIMO radar system consisting of the RadarLog baseband hardware and the frontend as it is displayed in Fig. 2.1.

![Figure 2.1: Fully functional FMCW MIMO radar system consisting of the frontend mounted on top of the baseband processing hardware RadarLog.](image)

The frontend in Fig 2.1 encompasses the TX and RX antennas as well as MMICs and other passive components. The frontend is easily mounted on top of the RadarLog via an RF connector on its back, which routes communication and control as well as downconverted receive signals between the two PCBs. The RadarLog baseband PCB employs a powerful FPGA to process the digitized signals and makes them available via a USB 3 interface.

To explain why the design of the system has been carried out the way it first was, the given requirements and the respective approach to satisfy them is discussed. The requirements are followed up with an investigation of the employed hardware on both
the frontend and baseband PCB, as well as by a short functional description of the system. The chapter closes by introducing the coordinate system and other relevant definitions that will be employed throughout this work.

2.1 Requirements

The potential of substrate integrated circuits as a substitute for the commonly employed microstrip technology became apparent throughout the last decade. However, the application of SIW in radar systems is still scarce. This work aims to create a frontend for a 77 GHz automotive radar system, utilizing the promising substrate integrated planar technology in cooperation with state of the art automotive radar MMICs from Infineon. Besides the employment of substrate integrated planar technology, a second main goal was to achieve a Sidelobe Level (SLL) suppression in the angular domain of approximately -40 dB within a limited field of view in the two-way pattern of the MIMO antenna array. This section provides insight into how these two goals were incorporated into the design.

2.1.1 Substrate Integrated Waveguide Design

The frontend of the radar system had to be manufactured on a predefined PCB layer stack with a single 127 μm layer of Rogers 3003 substrate. The height of the substrate is an essential factor for SIW design, and the predefined substrate height eliminated this degree of freedom. However, this was necessary because placement and routing of the MMIC components on the PCB is an art in itself and would have exceeded the scope of this thesis. Therefore, this project relied on an already available frontend PCB design that employed all necessary routing of the MMICs as well as connections for communication and power supply. The utilization of this template allowed to focus on the design of the antennas and their feed in more detail. The individual antenna elements employed on the frontend have been designed by creating a resonant array of slots within a SIW. The design of an antenna utilizing slots in the top conductive layer of a SIW has been studied for different applications and operating frequencies [13]. Analytical models describing the slot antenna behavior within waveguide structures have been derived in [14],[15], and successfully applied in design. However, the design of the antenna element is not straight-forward. The thin substrate and tight manufacturing tolerances impeded the design process, and considerable effort was invested in the numerical simulation of the structure to arrive at a functional antenna. The details of SIW, as well as the properties of slots within waveguides, are investigated in Chapter 3.

2.1.2 Sidelobe Level Suppression

SLL suppression in an array of antennas is usually done by applying a so-called taper function to shape the current distribution that is approximated by the array. A taper function is most commonly applied to the amplitude of individual antenna elements, which delivers the desired radiation characteristics. However, this reduces the total radiated power and requires fine-tuning of the individual antenna excitations. Low-cost integrated power amplifiers that are used to drive antennas in automotive radar applications are not designed to be fine-tuned to a specific gain, thus limiting the applicability of amplitude tapers. Another option for approximating a desired current distribution across the array aperture is to position equally excited antenna elements such that the element density corresponds to the desired taper function. This
approach is usually referred to as spatial density tapering and allows to shape the radiation pattern of the array while driving all antennas in equal and efficient working points [16]. Furthermore, the non-uniform spacing of antenna elements bypasses the spatial aliasing effect and mitigates the occurrence of grating lobes but also makes them harder to predict. However, the design of spatial density tapered arrays is more challenging, and since the employed SIW antenna is quite bulky, suitable design algorithms are scarce. It has been shown that the maximum SLL in a uniformly excited antenna array is given approximately by [17]

\[
SLL_{\text{dB}} = -10 \log \frac{N}{2} + 10 \log \left( 1 - \frac{\lambda_0}{2d_{\text{av}}} \right)
\]  (2.1)

where \( N \) is the number of elements in the array, \( \lambda_0 \) is the free space wavelength and \( d_{\text{av}} \) is the average element spacing. Therefore, the SLL is kept low by utilizing a large number of array elements with low average spacing between them, compared to the wavelength. The number of employed elements in this work with \( N_{\text{tx}} = 8 \) transmit antennas and \( N_{\text{rx}} = 16 \) receive antennas, together with the MIMO principle, allows synthesizing a virtual antenna array with \( N_v = N_{\text{tx}}N_{\text{rx}} = 128 \) antennas. Due to the bulkiness of the designed antenna element and the manufacturing tolerances, the minimum interelement distance is \( d_{\text{min}} \approx 0.72 \lambda_0 \) which, according to (2.1), gives a maximum SLL of \(-23.21\) dB. However, this result describes the optimal sidelobe level, which is defined in [17] as the level for which no sidelobe in the array factor will exceed the level of the first sidelobe. In this work a limited Field of View (FOV) is defined and the SLL within this FOV should be optimal while neglecting the SLL outside of the defined FOV. Deterministic design algorithms to create density tapered antenna arrays with a defined FOV exist and are investigated for linear and planar arrays respectively in [18] and [19]. However, these deterministic algorithms do not take the size of the antenna element or scan angles other than broadside radiation into account. These issues can be surmounted by resorting to iterative design algorithms that alter antenna element positions according to some kind of optimization like it is introduced in [20]. This work slightly alters the convex optimization approach introduced in [20] to obtain a two-way pattern with the required Sidelobe Level (SLL) suppression within the FOV.

### 2.1.3 Radar requirements

The requirements discussed above are the main focus of this thesis. However, they are not useful in defining the actual system parameters of the radar sensor. In order to not pick random system parameters, it is the self-proclaimed goal of the author to achieve a system that meets the requirements necessary for enhanced pedestrian protection as defined in [21]. Focusing especially on the requirements for the field of view in azimuth and elevation. However, no proof is delivered that these requirements are actually met.

### 2.2 Hardware

This section serves to introduce the employed hardware on the frontend and baseband PCBs respectively.
2.2.1 Front-End

The frontend PCB is composed of six individual routing layers that support the passive RF components like antennas and their feeds as well as routing for communication, control, and power supply. Furthermore, the frontend PCB is inhabited by the MMICs for waveform generation, power amplification, and downconversion of received signals, as well as the other lumped passive components and the connector to the baseband hardware.

Printed Circuit Board

The first two layers of the PCB are separated by a 127 µm thick RF substrate RO3003 [22], which has excellent dielectric loss properties and a stable permittivity versus temperature and frequency. The other routing layers are separated by prepreg and laminate layers made of the IS400 resin that is suited for demanding applications where thermal stability and reliability are a necessity [23]. Through-hole connections in the RF substrate are available with different sizes, but are most reliably manufactured with a diameter of 200 µm and a distance between two consecutive vias of 400 µm. The mechanical parameters of the frontend PCB are given in Tab. 2.1.

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<td>Dimension x-direction</td>
<td>160 mm</td>
</tr>
<tr>
<td>Dimension z-direction</td>
<td>120 mm</td>
</tr>
<tr>
<td>Weight</td>
<td>90 g</td>
</tr>
<tr>
<td>RF substrate</td>
<td>RO3003</td>
</tr>
<tr>
<td>RF substrate thickness</td>
<td>127 µm</td>
</tr>
</tbody>
</table>

Table 2.1: Mechanical parameters of the frontend PCB.

Active Components

Currently available integrated solutions for automotive radar applications rely on several individual MMICs for tasks like transmit waveform generation, downconverting of received signals, and power amplification. Although more compact solutions, with multiple TX and RX channels in one integrated circuit, exist [24], the radar system in this work builds upon such a distributed solution by utilizing the 77 GHz Radar frontend chipset from Infineon. The employed chipset distributes all the necessary tasks for active circuits to four MMICs. The arrangement of these chips on the frontend as depicted in Fig. 2.1 is explained with the schematic representation in Fig. 2.2. The RTN7745PL is a four-channel receiver which, in cooperation with the RRN7735PL three-channel transmitter form a scalable platform to build long- and mid-range systems. The transmitter and receivers are accompanied by the optional RPN7720PL dual power amplifier, which allows to increase the system output power plus additional output switching and phase switching of two TX channels. The RCC1010, called the RADAR companion, is a Complementary Metal-Oxide-Semiconductor (CMOS) chip which provides a fully digital interface to the RTN7735PL transmitter circuit. It features freely programmable modulation waveform generation and Phase Locked Loop (PLL) frequency control. All employed MMICs are inhabited by an Embedded Wafer Level Ball Grid Array (eWLB) package while the radar companion is housed in a TQFP package.
2.2. Hardware

![Diagram](image)

**Figure 2.2:** Arrangement of the RRN7735PL three-channel transmitter, the RTN7745PL four-channel receiver, the RPN7720PL dual power amplifier MMICs and the RCC1010 radar companion chip to form a radar system with eight transmit and sixteen receive channels.

![Diagram](image)

**Figure 2.3:** Schematic view of antenna positions.

**Passive Components**

The antenna element employed in the TX and RX array is the most critical component in the radar system. It consists of four longitudinal slots etched into the top wall of a shorted SIW that is also used to feed these slots. The details of the slot antenna and the arrangement of multiple slots in a shorted waveguide, called a resonant linear array, are further investigated in chapter 3. The SIW that feeds the TX antenna element is driven from the differential microstrip output of an RPN7720PL dual power amplifier. Since the package of the amplifier is not able to connect to a SIW directly, a transition between differential microstrip and SIW is necessary. A transition from microstrip is also necessary for the antennas in the RX array. However, the receiver RTN7745PL connects to the RX antennas via a single-ended microstrip line, which simplifies the necessary transition to the SIW. The waveform that is generated by the transmitter needs to be distributed to the power amplifiers, which further direct the signal to the desired TX antennas and to the receivers where it is used as a Local Oscillator (LO) to downconvert the received signals. Although, this work employs SIW, the signal distribution is done utilizing differential microstrip lines. The positions of the individual antennas are sketched in Fig. 2.3 and detailed positions are given in Table 2.3b and 2.3a respectively.
Table 2.2: Antenna positions. (a) TX (planar array, 2D). (b) RX. (linear array, 1D)

2.2.2 Base-Band

The frontend is designed to cooperate with the RadarLog from Inras and offers the option to change the configuration freely to the needs of the application at hand. RadarLog offers a USB 3 interface to a host, which can be used to configure the modes of operation for the FMCW radar system. The timing of the FMCW waveform as well as the FMCW ramp parameters can be programmed and the IF signals recorded in real-time. The main features provided by this versatile piece of hardware include [25]

- Sampling rates up to 65 Mega Samples per Second (MSPS) for up to 16 IF channels
- Ramp-synchronous sampling
- Arbitrarily programmable FMCW timing
- Binary phase shifting of output signals
- MIMO processing with arbitrary antenna activation
- Configurable signal processing

The RadarLog provides rapid prototyping capabilities, for radar sensors. As depicted in the block diagram in Fig. 2.4 it inherits a powerful Arria V FPGA with DDR3 memory and USB 3 connection to a receiving host. The system provides a clock signal and digital control signals to the frontend, connected via RF connectors, and digitizes up to 16 downconverted receive signals via two AFE5801 analog to digital converters.
2.3 Functional Description

The frontend is designed to enable MIMO radar processing for FMCW waveforms with multiple transmit antennas. The TX antennas can be activated arbitrarily by means of digital control signals in order to implement a virtual array. According to the block diagram in Fig. 2.2, an RTN7735PL in conjunction with the RCC1010 radar companion chip is used to generate the FMCW transmit signal. The activation sequence is programmed with the trigger and timing unit implemented in the FPGA of the RadarLog, which is further described below. The receive path on the right side of the frontend PCB is realized with four RRN7745PL receivers which perform the downconversion of the received signals. The Local Oscillator (LO) signal is provided to the receiver by the RTN7735PN transmitter and distributed through power amplifiers RPN7720PL. The remaining RF outputs of the RTN7735PN transmitter are used to feed the power amplifier which further connects to the individual transmit antennas. The configuration of the FMCW MIMO RADAR system is done entirely via a Matlab class that is provided by Inras and provides easy access to the RadarLog.

2.4 Preliminary

The discussion of basic theory and applied concepts in chapter 3 as well as the investigation of the design process in chapter 4 and the presentation of the measurement results in chapter 5 rely upon the utilization of Cartesian and spherical coordinates as shown in Fig. 2.5. The Cartesian coordinate system with spatial coordinates \(xyz\) is employed to define positions of individual antenna elements on the \(xz\) plane which is further also denoted as the antenna plane. The spherical coordinate system with radius \(r\), elevation angle \(\theta\) and azimuth angle \(\phi\) is used for the representation of antenna radiation patterns and radar target positions. In order to keep notation and naming simple during the design of the array topology two planes are defined. Namely the azimuth plane (\(\theta = \pi/2, \phi \in [0 \pi]\)) and elevation plane (\(\phi = \pi/2, \theta \in [0 \pi]\)). Furthermore, the direction orthogonal to the antenna plane (parallel to the \(y\) axis) will be denoted as the broadside direction, and directions parallel to the antenna plane will be called endfire. A word on notation, the norm of a vector \(\mathbf{r}\) is written as \(|\mathbf{r}|\) or \(\mathbf{r}\) in case \(r\) is not suitable. The unit vector of \(\mathbf{r}\) is represented as \(\hat{\mathbf{r}}\).
Figure 2.5: Coordinate system definition. (a): Cartesian coordinate system with indication of antenna plane. (b): Spherical coordinate system.
Chapter 3

Theory and Applied Concepts

This chapter introduces the concepts of MIMO radar, antennas and planar antenna arrays, as well as the SIW and slot antennas in waveguides, which are necessary to accurately and efficiently describe the design process investigated later in chapter 4 and the measurements presented in chapter 5. The first section introduces radar in general before focusing on the FMCW system operation and efficient means to compute target parameters like range, radial velocity, and Angle of Arrival (AoA). The properties and benefits of the MIMO concept are explained by discussing the enhancement of angular resolution compared to a SIMO system. Furthermore, the standard schemes to achieve orthogonal transmit signals and the calibration of multi-channel radar systems is discussed. In the second part a mathematical investigation of antennas and antenna arrays is delivered as a basis for the comparison of MIMO array topologies, and the introduction of an algorithm used to design spatial density tapered arrays with a convex optimization approach. The third section has its focus on the radiation mechanism of slot antennas within a rectangular waveguide, and further investigates the design of an array of such radiators within the broad wall of said waveguides. The last section of this chapter deals with the SIW, its principle of operation, and its loss mechanisms.

3.1 Radar

Modern radar systems are sophisticated transducer/computer systems that are employed to detect, track, image, and classify in a wide range of military, industrial, and civilian application scenarios [26]. This section first delivers a rough classification of radar systems based on antenna configuration, transmitted waveform, and operating frequency. Furthermore, the FMCW radar principle in terms of linear frequency modulation is introduced, and the extraction of target parameters such as range, Doppler velocity, and angle of arrival is explained. Finally, the concept of MIMO radar and its benefits will be discussed.

3.1.1 Classification of Radar Systems

The wide range of different radar applications leads to a number of system concepts operating in a variety of frequency bands.
Antenna Configuration

There are two basic antenna configurations for radar applications, namely quasi mono- and multi-static systems. A radar system is considered to be mono-static if the spatial position of the transmit and receive antenna is (almost) identical. This means that either only one antenna is used as a transceiver or individual antennas for transmitting and receiving are spaced so close that the range and angle of a potential target are nearly identical for both antennas. For multi-static radar systems, the range and the angle to the target are significantly different for each antenna [26]. The MIMO radar with co-located antennas is a particular case of multi-static radar systems since it does not exploit the different illumination angles from the individual transmit antennas to increase performance.

Transmitting Waveform

It is possible to distinguish between two general modes of radar operation, namely Contious Wave (CW) and pulsed systems, which are further divided into several subcategories

In pulsed systems, a short EM pulse is emitted while the receiver is isolated from the transmitter to protect the sensitive receiver components from the high peak power during pulse transmission. For the main proportion of the duty cycle, the receiver listens for echoes that may have been reflected from objects in the observation environment. The periodicity of the duty cycle is called the Pulse Repetition Interval (PRI) and determines the unambiguous range of the system. One can further classify pulsed radar systems by the coherency of the phase relationship of two consecutive pulses, which is essential to extract Doppler information from the received echoes [26]. Furthermore, sophisticated pulses with internal frequency or phase modulation in combination with pulse compression techniques can be used to enhance the performance of pulsed radar systems further as it is, for example, used in [27].

In a CW radar system the receiver and transmitter are usually active without interruption, which implies a non-negligible coupling between the receive and transmit parts of the system. Thus, most CW systems are bi-static or low power mono-static configurations used for short-range applications. The main advantage of (unmodulated) CW radars is the ability to handle any conceivable velocity of targets at any range, while with pulse Doppler radars this is only possible with considerable effort [26]. Historically the advantages of a continuous wave system were recognized early, but due to limited application scenarios and technological limitations at the time, the CW systems only played a minor role in the early days of radar systems. An unmodulated (monochromatic) CW system is fundamentally not capable of unambiguously measuring range, since there is no way of extracting the round trip time of the received signals. Modulation of the frequency introduces range measurement capability, but one has to accept some compromises, such as ambiguous range and ambiguous Doppler velocity, that are also the bane of pulsed radar systems.

The main disadvantages of pulsed radar systems are the low duty cycle, and the large ratio between peak and average transmitted power. This is not the case for CW systems, allowing the use of cheaper off-the-shelf low power electronic components, which is especially desirable for automotive systems due to the low cost and high
yield requirements. However, the low peak power of CW severely limits the energy on target return and CW compensates this with a longer illumination duration.

Operating Frequency

The radio part of the electromagnetic spectrum is usually considered to be the frequency range from 30 Hz to 300 GHz. Different naming conventions exist for sub-bands within the radio spectrum. In general the ITU\textsuperscript{1} standard, that divides the radio spectrum into 12 bands [28] is commonly used. However, radar systems are classified with a naming convention established by the US Institute of Electrical and Electronic Engineers (IEEE) which is defined in [29].

3.1.2 Radar Equation

The single most powerful description of a radar system’s performance is given through the radar equation, which describes the power $P_{rx}$ received from a target echo in terms of system and target parameters. A simple form to write the radar equation for a mono-static system is

$$P_{rx} = \frac{P_{tx} G_{tx} RCS_t G_{rx} \lambda_0^2}{4\pi R^2}$$  \hspace{1cm} (3.1)

with the power $P_{tx}$ transmitted by the radar system, $G_{tx}$ and $G_{rx}$ the gain of transmit and receive antenna respectively and $\lambda_0$ the free space wavelength. The first term in (3.1) describes the power density at distance $R$ due to the power radiated from the transmit antenna. The numerator of the second term describes the radar-cross-section of the target while the denominator accounts for the path loss of the echo on its return path. The third term in (3.1) represents the effective antenna aperture of the receive antenna. The radar equation offers simple predictions about target ranges and is usually used as a powerful tool for radar system analysis and design by relating the received power to the noise power seen in the receiver.

3.1.3 FMCW Principle

Many applications with target ranges below a few hundred meters employ CW radar units. Although the history of FMCW dates back to 1920 [30], its use was limited until a detailed investigation for linear frequency ramps was carried out in [31]. In the following, the operational principle for a linear FMCW radar system is explained, and parameter extraction for a single target is investigated.

Operational Principle

Fig. 3.1 depicts a simplified block diagram of the analog stage of an FMCW radar system with one transmit and one receive antenna, and a single target in the observation scene. The heart of an FMCW system is a tunable RF source which allows the generation of signals with arbitrary frequency. For the high frequencies used in this work, this is usually done by driving a Voltage Controlled Oscillator (VCO) with a Phase Locked Loop (PLL) and later use frequency multipliers to achieve the desired output frequency [32]. One part of the generated signal $s_{tx}$ gets directed towards an optional Power Amplifier (PA) and is radiated by the Transmitter (TX) antenna,

\textsuperscript{1}The International Telecommunication Union (ITU) was founded in 1865 to facilitate international connectivity in communications networks. It allocates global radio spectrum and satellite orbits, develops the technical standards which ensure that networks and technologies seamlessly interconnect, and strives to improve access for underserved communities worldwide.
while the other part is directed towards the receive mixer to be used as a Local Oscillator (LO) signal. The transmitted wave gets reflected by the target, and experiences a delay over twice the target range before the echo gets captured by the Receiver (RX) antenna. After an optional Low Noise Amplifier (LNA), the received signal \( s_{rx} \) is fed into the receive mixer’s RF input resulting in the signal \( s_{mix} \) at the Intermediate Frequency (IF) port of the mixer. Prior to discretization a lowpass filter is applied to the output of the mixer to get rid of unwanted high frequencies in the \( s_{mix} \) signal, resulting in the beat signal \( s_{beat} \) which serves as basis for further processing. The basic operation principle of linear FMCW systems is further explained with the aid of Fig. 3.2. It depicts the frequency of the signals \( s_{tx} \) and \( s_{rx} \) as a function of time, assuming a triangular modulation in the interval \( [f_0, f_0 + B_{chirp}] \) for two targets, one static and one moving at the same range. The beat signal \( s_{beat} \) contains both information about the target range as well about target velocity. For a static target, the beat frequency is proportional to the range while for a moving target, the beat frequency experiences a shift proportional to the radial velocity of the target. In case of moving targets, it is necessary to transmit at least two signals with different ramp slopes to determine both the targets range and its velocity [33]. This is usually done by combining linear modulations with both positive and negative slopes, so called up- and down-chirps.
Derivation of the Beat signal

Considering a single up-chirp modulation with bandwidth $B_{\text{chirp}}$ in the frequency range $[f_0, f_0 + B_{\text{chirp}}]$ with duration $T_{e,\text{up}}$, the transmit signal contains a time-dependent phase $\alpha_{\text{tx}}$ and is written as

$$s_{\text{tx}} = A_0 e^{j\alpha_{\text{tx}}(t)}$$

with

$$\alpha_{\text{tx}}(t) = \int_0^t 2\pi f(t) \, dt = 2\pi \left(f_0 + \frac{B_{\text{chirp}}}{2T_{e,\text{up}}}ight) t$$

where $t$ denotes continuous time and $A_0$ represents the amplitude of the transmit signal after optional power amplification. The radiated wave gets reflected from a target at distance $R_t$ and the echo is captured by the receive antenna. After optional low noise amplification, the received signal is written as

$$s_{\text{rx}} = A_0 \Psi e^{j\alpha_{\text{rx}}(t)}$$

with $\alpha_{\text{rx}}(t)$ denoting the time dependent phase of the receive signal and $\Psi$ accounting for the gain of the TX and RX antennas, gain of the PA and LNA, the free space propagation loss, the atmospheric attenuation, and the RCS of the target \(^2\). Assuming no additional phase shifts caused by the antennas, the LNA, and the target reflection, the transmitted and received signals are related by

$$s_{\text{rx}} \propto s_{\text{tx}} (t - \tau_{\text{TOF}})$$

with $\tau_{\text{TOF}} = 2R_t/c_0$ representing the round-trip time due to propagation over twice the target distance $R_t$. Therefore, the phase of the received signal can be written as

$$\alpha_{\text{rx}}(t) = 2\pi f_0 \left(t - \frac{2R_t}{c_0}\right) + \pi \frac{B_{\text{chirp}}}{T_{e,\text{up}}} \left(t - \frac{2R_t}{c_0}\right)^2.$$  

(3.6)

The transmit and receive signals are fed into the LO and RF port of the receive mixer respectively. Assuming an ideal mixer the signal at the IF port is

$$s_{\text{mix}} = \Re \{s_{\text{tx}}s_{\text{rx}}\}$$

which basically states that the IF signal is proportional to the product of the cosines of the input signal phases

$$s_{\text{mix}} \propto \cos(\alpha_{\text{tx}}(t)) \cos(\alpha_{\text{rx}}(t)).$$

(3.8)

Eq. (3.8) implies that the output signal contains the sum and difference frequencies of the input signals $s_{\text{tx}}$ and $s_{\text{rx}}$. The output of the receive mixer is fed into a low-pass filter to get rid of the unwanted high-frequency parts and gives the so-called beat-signal $s_{\text{beat}}$ while at the same time performing anti-aliasing before digitization. The dominant frequency of the beat signal for a single target at range $R_t$ is

$$f_{\text{beat}} = \frac{1}{2\pi} \frac{d}{dt} |\alpha_{\text{tx}} - \alpha_{\text{rx}}| = \frac{2B_{\text{chirp}}R_t}{c_0T_{e,\text{up}}}.$$  

(3.9)

Eq. (3.9) represents the elegant conclusion of linear FMCW systems, being that the dominant frequency of the beat signals is proportional to the target range and the

\(^2\)The value of $\Psi$ is modeled with the radar range equation
analysis is straightforward utilizing spectral analysis of the beat signal. The beat signal is now the basis for the further extraction of target parameters like range and velocity and, for the case that multiple RX antennas are employed, Angle of Arrival (AoA).

**Target Range**

The beat signal for a single target is basically a sinusoid truncated by the chirp duration and its corresponding spectrum is a sinc-function centered around the beat frequency as it is visualized with the help of Fig. 3.3. The beat spectrum can be easily interpreted as a range profile through a transformation of the frequency axis by rearranging (3.9) into

$$R_t = \frac{c_0 T_{c,up} f_{\text{beat}}}{2B_{\text{chirp}}}. \quad (3.10)$$

Assuming the chirp duration $T_{c,up}$ is much larger than the round trip time $\tau_{\text{TOF}}$ the spectrum of the beat signal is resolvable to an accuracy of $2/T_{c,up}$ Hz (distance between zero crossings). However, it is common practice to define the resolution bandwidth $\Delta f_{\text{beat}}$ between its $-3$ dB (half power) points which corresponds to $1/T_{c,up}$ Hz (if no window function is applied). Substituting this resolution bandwidth into (3.10) gives the range resolution

$$\delta R = \frac{T_{c,up} \Delta f_{\text{beat}}}{2B_{\text{chirp}}} = \frac{c_0}{2B_{\text{chirp}}} \quad (3.11)$$

which is inversely proportional to the chirp bandwidth. The maximum beat frequency is determined by the cut-off frequency of the employed low-pass filter $f_{c,\text{LPPF}}$ which is usually selected to be $f_s/2$, with $f_s$ denoting the sampling frequency of the ADC. Therefore, the maximum detectable target range is given by

$$R_{\text{max}} = \frac{T_{c,up} f_{c,\text{LPPF}}}{2B_{\text{chirp}}}. \quad (3.12)$$
Target Velocity

Up until now, the beat signal has been studied for stationary targets. The Doppler effect, as described in [34], will cause a frequency offset

\[ f_{\text{doppler}} = \pm \frac{2v_r f(t)}{c_0} \]  

(3.13)

in the reflected wave, with \( v_r \) denoting the radial velocity of the target. Since this frequency offset is also visible in the beat signal this effect will cause a displacement in range

\[ \Delta R = \pm \frac{T_{\text{c,up}} f_{\text{doppler}}}{2B_{\text{chirp}}} . \]  

(3.14)

This ambiguity described by (3.14) can be resolved by combining up- and down-chirps to a so-called triangular modulation, as shown in Fig. 3.2. However, it is more widely employed to compare consecutive range measurements and calculate the target velocity from these sequential measurements. In modern FMCW signal processing, the combination of multiple chirps is called a cycle, and for short chirp durations it is safe to assume that the target velocity does not significantly change throughout multiple chirps. The maximum unambiguous Doppler velocity is then determined by the chirp repetition rate. The processing of such cycles in terms of two-dimensional spectral analysis delivers range as well as velocity for targets within the observation scene.

Angle of Arrival

Obtaining target parameters like range and Doppler velocity is easily done with a single transmitter and receiver pair as it is shown above. However, in many applications this is not sufficient. Automotive applications, for example, demand an understanding of the road environment, which implies the mapping of target echoes onto a two dimensional (2D) or even three dimensional (3D) coordinate grid. With a single TX/RX antenna pair, this information is not available. By employing a SIMO radar as depicted in Fig. 3.4, it is possible to estimate the AoA of the echo impinging on the radars receive channels by evaluating the progressive phase shift in the individual received signals. With the range and angle information, it is possible to map the echo onto a 2D or 3D coordinate system. Estimating the angle of arrival of an object requires a setup with at least two RX antennas separated by a known distance \( d_{\text{rx}} \) as shown in Fig. 3.4. For simplicity, it is assumed that the impinging wave is sufficiently approximated by a plane wave. Due to the incident angle \( \theta_i \) of the plane wave, the signal has to travel an additional distance of \( d_{\text{rx}} \sin(\theta_i) \) to the second
receiver, resulting in a phase difference of

\[ \Delta \alpha_{\text{rx}} = \frac{2\pi}{\lambda_0} d_{\text{rx}} \sin(\theta_i). \] (3.15)

In general, a radar system employs \( N_{\text{rx}} \) receive antennas, and the received signal at each subsequent antenna has an additional phase shift of \( \Delta \alpha_{\text{rx}} \) relative to the preceding antenna. Due to this linear progression of phase shifts across the receive array aperture, the phase shift \( \Delta \alpha_{\text{rx}} \) can be reliably estimated through Fourier processing of the sampled signal across the receive array. Increasing the number of antennas results in more spatial samples and thus an improved angular resolution. Furthermore, the larger the distance between the antenna elements, the better the angular resolution will be. However, a \( d_{\text{rx}} \) larger than \( \lambda_0/2 \) might introduce ambiguities since one value of \( \Delta \alpha_{\text{rx}} \) does not correspond to a single value for \( \theta_i \) anymore. The angular resolution for a linear array with \( N_{\text{rx}} \) elements and interelement distance \( d_{\text{rx}} \) is

\[ \delta \theta_{\text{inc}} = \frac{\lambda_0}{(N_{\text{rx}} - 1) d_{\text{rx}} \cos(\theta_i)}. \] (3.16)

The denominator in (3.16) is called the effective aperture size of the array which with the cosine of the incident angle \( \theta_i \) resulting in worsening of the angular resolution for large incident angles.

Other target parameters available from the echo are size, shape, and also tangential velocity. However, this work confines itself to the three primary parameters most relevant to automotive applications that were introduced earlier.

3.1.4 MIMO Radar Basics

Automotive radar sensors should obtain high resolutions for all parameters to be able to identify objects clearly within the road environment. During the discussion of the AoA principle, in Sect. 3.1.3, it was concluded that high range resolution is achieved by deploying high modulation bandwidths, while large effective antenna array apertures yield a satisfying angular resolution. The increase in array aperture is often not feasible since additional channels, space, and hardware is required. Radar, in the context of a MIMO system model, offers increased spatial resolution and sensitivity compared to a SIMO system. The main characteristic of a MIMO radar is that it employs multiple transmit antennas that emit independent waveforms [8] as depicted in Fig. 3.5. The individual waveforms are chosen such that their superposition can be separated during the processing of the received signals. This method allows the generation virtual antennas and increase the array aperture without increasing the number of physical antennas. This additional information leads to numerous advantages such as the mitigation of heavily angle-dependent radar cross sections [35] and an increased number of resolvable targets. The MIMO radar provides more degrees of freedom, which leads to improved angular resolution and target parameter identifiability [8]. Consider a SIMO system utilizing one transmit and \( N_{\text{rx}} \) receive antennas with interelement spacing \( d_{\text{rx}} \) as seen in Fig. 3.6. The resulting angular resolution of the SIMO system can be calculated using (3.16) which implies that a twofold increase in angular resolution would require twice as many receive antennas or the doubling of the spacing between elements. Interelement spacing larger than half a wavelength
3.1. Radar

**Figure 3.5:** Visualization of the MIMO radar principle. Multiple transmit antennas transmitting orthogonal (different colors) waveforms that are separable in the individual receive channels.

**Figure 3.6:** Angle of Arrival Estimation with a SIMO vs. MIMO radar system that both achieve the same angular resolution.

results in ambiguities in the incident angle and for each added receive antenna, an additional processing chain (LNA, mixer, low-pass and ADC) is necessary. Therefore, none of the methods to increase the angular resolution for SIMO systems is desirable. However, using MIMO concepts, a twofold increase in angular resolution can be achieved with just one additional transmit antenna, as is explained with the help of Fig. 3.6. In Fig. 3.6 a SIMO system with four receive antennas is shown. A target reflection impinging on the receive array results in a progressive phase shift of $\Delta \alpha_{\text{rx}}$ in the four received signals. Fig. 3.6 depicts a MIMO system with two transmit and two receive antennas. A signal emitted from TX0 would also cause a progressive phase shift of $\Delta \alpha_{\text{rx}}$ in the two received signals and the same is true for a signal emitted from TX1. However, due to the distance between the two transmitters $d_{\text{tx}}$, the phase of a signal emitted from TX1 would experience an additional shift of $2\Delta \alpha_{\text{rx}}$ concerning the received signals originated from TX0. Therefore, if the signals from TX0 and TX1 are separable during the processing of the individual receiver channels, the four signals show a progressive phase shift of $\Delta \alpha_{\text{rx}}$ and can be treated as originating from

---

3For element spacings larger than half a wavelength the mapping between progressive phase shift and incident angle is not a bijective function anymore.
a receive array with four elements like in the SIMO case before. The MIMO configuration of two transmit and two receive antennas therefore synthesizes a virtual array of four antennas. The angular resolution of the SIMO and MIMO radar system are equal, although the MIMO configuration only employs four antennas instead of five like the SIMO system. In general, it can be said that a MIMO radar with $N_{tx}$ transmit and $N_{rx}$ receive antennas, and with proper antenna placement, can synthesize a virtual antenna array with $N_v = N_{tx}N_{rx}$ individual elements. Since the benefits of a MIMO system arise from the orthogonality of its transmitted waveforms, it is evident that great care has to be taken in the design of these waveforms. Orthogonality in the transmitted signals can be achieved by the following methods.

- With Code Division Multiple Access (CDMA), it is possible to obtain excellent real-time measurement capability due to the simultaneous transmission of coded waveforms on all transmit channels on the total available bandwidth. The signals are separable in the receive channels by using code sets with low cross-correlation values. CDMA-MIMO radars also deliver high interference robustness. However the price for that excellent performance is paid with complex hardware requirements that are necessary for generating and processing fast coded signals.

- Frequency Division Multiple Access (FDMA) too, allows the simultaneous transmission on all transmit channels. However, the individual channels are separated by the division of the available bandwidth into subcarriers. The more transmitters are employed, the less bandwidth is available per channel, which has a negative impact on range resolution, ambiguity or Doppler-ambiguity.

- Time Division Multiple Access (TDMA) is the simplest form of achieving orthogonality between transmitted waveforms and is commercially used in most automotive radar systems nowadays. This modulation scheme does not transmit on all TX channels simultaneously but activates individual transmitters sequentially. The sequential probing of the environment results in a significantly increased cycle time but introduces ideal orthogonality and allows the use of more straightforward and thus cheaper hardware. Therefore, TDMA-MIMO is especially suited for automotive radar sensors due to the low cost requirement and large-scale production numbers.

### 3.1.5 Radar Calibration and Mutual Coupling Compensation

Error sources for FMCW MIMO radar systems include channel gain and phase errors, as well as different lengths in feed lines and mutual coupling. Such errors are often unavoidable and significantly degrade the performance of the radar system by impacting the quality of the beam pattern and deteriorating the AoA estimation accuracy. Considering a MIMO FMCW radar system with $N_v$ virtual antenna elements positioned at $r_{x,n}(n), n = 1, \ldots, N_v$, with $r_{x,n}(1) = 0$, as schematically depicted in Fig. 3.7. The operational principle to arrive at the beat-signal $s_{\text{beat}}$ was already discussed above. The range profile for each chirp is calculated from the discretized beat signal through a discrete Fourier transform (DFT) resulting in the range profiles $s_r(h,t')$ where $h$ denotes the discrete range bins and. A common mathematical model for this signal, also known as range-compressed signal, using vector notation can be written as $[36]$

\[
s_{r,\text{id}} = \begin{bmatrix} s_{r,\text{id}}(h,1) & s_{r,\text{id}}(h,2) & \cdots & s_{r,\text{id}}(h,N_v) \end{bmatrix}^T
\]  

(3.17)
with
\[
s_{t,\text{id}}(h, n) = A_0 \Psi W(h-R_t) e^{j2\pi \frac{\nu_0}{f_0} r_{x,n}(h) \cos \phi_t} .
\]  
(3.18)

where \( W \) corresponds to the Fourier transform of the temporal window function during the up-chirp. The range profiles are further analyzed by a beamforming algorithm to perform AoA estimation. However, this idealized signal model does not account for coupling between the individual channels expressed by the coupling factors \( c_{ij} \), \( i = 1, \ldots, N_v \), \( j = 1, \ldots, N_v \) and the exact channel gain and phase values. In order to model linear, target angle-independent coupling, these coupling factors can be described in terms of an \( N_v \times N_v \) coupling matrix \([C]\) which relates the idealized signal model to the actual measurement
\[
s_t(h) = [C] s_{t,\text{id}}(h) .
\]  
(3.19)

The effects of mutual coupling can be mitigated by introducing the transformation matrix \([T]\). However, appropriate choices for \([T]\) are not easy to find. One way to do this is to set up a least squares problem [37] by compiling \( N_p \) measurements into
the matrix

\[ [S_t] = \begin{bmatrix} s_{r,1}(h_1) & s_{r,2}(h_2) & \cdots & s_{r,N_p}(h_{N_p}) \end{bmatrix} \]

and for these measurements composing the signal models into the matrix

\[ [S_{r,\text{id}}] = \begin{bmatrix} s_{r,\text{id},1}(h_1) & s_{r,\text{id},2}(h_2) & \cdots & s_{r,\text{id},N_p}(h_{N_p}) \end{bmatrix}. \]

The resulting optimization problem can be written as

\[ [T] = \arg \min_{[T]} \|[S_{r,\text{id}}] - [T][S_t]\|_F, \]

with \( \| \cdot \|_F \) denoting the Frobenius norm, and has a known solution given through

\[ [T] = [S_{r,\text{id}}][S_t]^H \left( [S_t][S_t]^H \right)^{-1} \]

with \( \cdot^{-1} \) denoting the inverse and \( \cdot^H \) corresponding to the conjugate transpose of a matrix. The problem is to create well-defined measurement scenarios that correspond accurately to the signal model. For high frequency radars this means knowledge of target positions and antenna phase centers with sub-millimeter accuracy. Several approaches were introduced that aim to relax the requirements on the calibration measurements and this work investigates the performance of two calibration methods. The first method is very simple and relies on a single measurement of a stationary target at broadside as depicted in Fig. 5.9 and is referred to as vector calibration. The second method, introduced in [36], utilizes measurements of multiple stationary targets at known angles and compares them to an idealized signal model which will further be called matrix calibration. Besides the complexity and effort that has to be considered for the two calibration approaches, their main difference is that the second method effectively accounts for mutual coupling effects between array elements and therefore, is expected to be valid over a wider range of target angles.

Vector Calibration

The vector calibration method obtains a single complex calibration coefficient \( t_{nn} \) for each of the \( N_v \) virtual array elements which are positionend in the main diagonal of the transformation matrix \([T]\). Therefore, coupling coefficients \( c_{ij} \) for \( i \neq j \) are assumed to be zero, hence the coupling between antennas is not accounted for. The measurement result in each channel should correspond exactly to the signal model

\[ t_{nn} s_r(h, n) = t_{nn} c_{mn} s_{r,\text{id}}(h, n), \]

thus the calibration coefficients are calculated with

\[ t_{nn} = \frac{s_{r,\text{id}}(h, n)}{s_r(h, n)}. \]

This approach, based on a single measurement \((N_p = 1)\), accounts for constant gain and phase errors due to differences in the integrated power amplifiers, receivers, and differences in feed lengths.

Matrix Calibration

In contrast to the vector calibration technique described above, a matrix calibration that accounts for coupling between channels requires multiple measurements
in a well defined measurement scenario to obtain results that are coherent with the employed signal model. The sub-millimeter accuracy required for target positioning during measurements disqualifies this calibration technique for many applications. In [36] a calibration approach for FMCW MIMO radars was introduced that accounts for mutual coupling and requires precise angular positioning of targets. However, it relaxes the accuracy requirements on target positions with respect to the antenna phase centers. This is done by making the signal model $s_{t,p}$ dependent on the obtained measurements and solving the optimization problem (3.23). A well-specified optimization problem requires $N_p \geq N_v$ independent measurements

$$s_{t,p}(h) = [s_{t,p}(h,1) \ s_{t,p}(h,2) \ \cdots \ s_{t,p}(h,N_v)]^T, \ p = 1, \cdots, N_p$$

(3.26)

with radar targets at known angular positions $\phi_{t,p}$ and $\theta_{t,p}$. The known angular position of the target allows the composition of an ideal signal model based on the measurement of the first channel

$$s_{t,1,d,p}(h) = s_{t,p}(h,1) \left[ 1 \ e^{j\varphi(2,\phi_{t,p},\theta_{t,p})} \ \cdots \ e^{j\varphi(N_v,\phi_{t,p},\theta_{t,p})} \right]^T$$

(3.27)

with

$$\varphi(n,\phi,\theta) = 2\pi \frac{f_0}{c_0} \left[ r_{x,n}(n)\cos\phi + r_{z,n}(n)\cos\theta \right].$$

(3.28)

The signal model in (3.27) sets the amplitude and phase of the first antenna equal to the measurement for channel $n = 1$ and then subsequently calculates the expected relative phases in the other channels. The resulting matrices composed according to (3.20) and (3.21) are subject to the optimization problem posed in (3.23).

Both methods are successfully demonstrated in chapter 5.

### 3.2 Antennas and Antenna Arrays

For wireless systems, the antenna is one of the most critical components. A well-designed design of the antenna can relax system requirements and improve overall system performance [38]. An antenna serves as a transducer between EM waves that are propagating through free space and EM waves that are guided within a transmission line. The antenna provide to an electrical system the same functionality as the eye and ear provides to a human. Furthermore, as eyes and ears are strategically placed on a human’s head to provide acoustic localization capabilities and depth perception, multiple individual antennas can be strategically arranged in a so-called array to increase directional properties and add beam steering capabilities. This section introduces antenna properties and antenna array characteristics, as well as a mathematical description of their radiative properties and methods to manipulate their radiation. Furthermore, different MIMO array topologies and their suitability for fabrication with planar transmission line technologies are investigated.

#### 3.2.1 Antenna Principles

An antenna is an interface between guided EM waves and EM waves that are propagating in free space. Therefore, an antenna is described with two different sets of properties, one set for the interface towards the transmission line and one set for the interface towards free space. The transmission line point of view deals with an equivalent circuit model and parameters such as impedance and reflection coefficient.
while the free space point of view models an equivalent current distribution based on the geometry of the antenna and derives parameters like the radiation pattern and directivity. In the following, both of these viewpoints will be investigated by studying the slot antenna in an infinite ground plane, fed by a Coplanar Waveguide (CPW) as depicted in Fig. 3.8(a).

**Equivalent Circuit Point of View**

Assuming the CPW connecting to the slot antenna in Fig. 3.8(a) is lossless and described with a characteristic impedance $Z_{CPW}$ as well as a propagation constant $\beta_{CPW}$ and excited by a source\(^4\) with impedance $Z_G$. The slot antenna is described by the impedance

$$Z_A = R_A + jX_A = R_I + R_r + jX_A. \quad (3.29)$$

for which usually both real and imaginary parts are functions of frequency. Where $R_A$ corresponds to the total radiated power (radiation resistance $R_r$) and the power dissipated into heat (loss resistance $R_l$) due to finite conductivity and the dielectric filling of the antenna. The imaginary part of (3.29) corresponds to the total power stored in the reactive field around the antenna. The Thevenin equivalent circuit representation of source, transmission line and antenna is given by Fig. 3.8(b).

Ideally, all the power made available by the source is delivered to the radiation resistance $R_r$. However, two mechanisms prevent maximum power transmission. On the one hand, power might be reflected due to impedance mismatches between the generator, transmission line, and antenna, resulting in standing waves and energy storage in the line. On the other hand, the power delivered to the antenna partially dissipates into heat due to naturally occurring finite conductivity and dielectric losses in the antenna\(^5\) represented by the resistance $R_l$.

\(^4\)Antenna is described in transmit mode. Due to the reciprocity of antennas, the analysis is also valid for receive mode operation.

\(^5\)Power dissipation into heat also occurs in the transmission line. However, for simplicity a lossless transmission line is assumed.
Based on the investigation of generator and load mismatches in [39] it is concluded that maximum power is transferred to the load if the conjugate matching condition is satisfied. Assuming the transmission line is matched to the generator \( Z_G = Z_{CPW} \), conjugate matching is satisfied if

\[
Z_G = Z_A^*.
\]  

(3.30)

The undesired effects due to impedance mismatch and power dissipation in the antenna are summarized by the antenna efficiency \( \epsilon_0 \) that relates the power radiated by the antenna with the power incident on the antenna input. The efficiency is generally written as

\[
\epsilon_0 = \epsilon_{cd} \epsilon_r
\]  

(3.31)

with \( \epsilon_r = (1 - |\Gamma_V|^2) \) describing the power reflected due to impedance mismatch (with \( \Gamma_V \) denoting the voltage reflection coefficient) and \( \epsilon_{cd} \) describing the combined conductive and dielectric efficiency\(^6\).

**Free Space Point of View**

An antenna radiation pattern is defined as "a mathematical function or graphical representation of the radiation properties such as power flux density or field strength, of the antenna as a function of space coordinates" [40]. The modeling of an antenna's radiation properties is an electromagnetic boundary-value problem for which the solution describes the EM field generated by the antenna. The field created by the antenna divides into reactive and radiative components. While the reactive fields store energy and are only dominant near the antenna, the radiative fields carry energy away from the antenna in the form of EM waves and are dominant far away. Therefore, the space surrounding an antenna is often divided into subregions, namely the reactive near-field, the radiative near-field, and the far-field region of the antenna [40] as seen in Fig. 3.9. The computation of the field around the antenna is quite complicated. However, within the radiative near-field and far-field regions, some useful approximations may be applied, allowing a more straightforward analysis of the radiated field. The boundaries of the subregions are shown in Fig. 3.9, and defined in terms of the maximum antenna size \( D_{ant} \) and the operational wavelength \( \lambda_0 \).

The fields created by the antenna have to satisfy Maxwell’s equations which, for a lossless, homogeneous and isotropic medium and time-harmonic fields, are written as [41]

\[
\nabla \times E = -M - j\omega \mu_0 H, \quad \text{(3.32)}
\]

\[
\nabla \times H = J + j\omega \varepsilon_0 E, \quad \text{(3.33)}
\]

\[
\nabla \cdot E = \frac{q_e}{\varepsilon_0}, \quad \text{(3.34)}
\]

\[
\nabla \cdot H = \frac{q_m}{\mu_0}. \quad \text{(3.35)}
\]

In (3.32)-(3.35) both electric (\( J \)) and magnetic (\( M \)) current densities and electric (\( q_e \)) and magnetic (\( q_m \)) charge densities represent the sources\(^7\) of the excited electric (\( E \))

\(^6\)These efficiencies are usually combined because both dissipate energy into heat and it is hard to account for them separately.

\(^7\)Although magnetic sources do not exist they are often introduced as electrical equivalents to facilitate solutions of boundary value problems.
and magnetic ($\mathbf{H}$) fields. The sources are related to one another by the continuity equations

\begin{align}
\nabla \cdot \mathbf{J} &= -j\omega q_{e1}, \\
\nabla \cdot \mathbf{M} &= -j\omega q_{m}.
\end{align}

(3.36)  
(3.37)

In addition to satisfying (3.32)-(3.35), the resulting fields must satisfy the radiation condition\footnote{The radiation condition requires that the waves traveling outwardly from the source in an infinite homogeneous medium vanish at infinity.} and the boundary conditions of

\begin{align}
-\hat{n} \times \mathbf{E}_d &= \mathbf{M}_s, \\
\hat{n} \times \mathbf{H}_d &= \mathbf{J}_s, \\
\hat{n} \cdot (\varepsilon_0 \mathbf{E}_d) &= q_{e,s}, \\
\hat{n} \cdot (\mu_0 \mathbf{H}_d) &= q_{m,s}.
\end{align}

(3.38)  
(3.39)  
(3.40)  
(3.41)

Where $\hat{n}$ denotes the unit vector perpendicular to the boundary surface and the subscript $d$ symbolizes the difference of the fields across the boundary. Eq. (3.38) and (3.39) impose the boundary conditions on the discontinuity of the electric and magnetic tangential components and (3.40) and (3.41) force the boundary conditions to hold for discontinuities in the normal components of the electric and magnetic flux densities.

To solve a radiation problem, the first step is to model an equivalent electric ($\mathbf{J}_{eq}$) and magnetic ($\mathbf{M}_{eq}$) current distributions as sources for the generated fields. For
aperture antennas, this is done using the field equivalence principle\(^9\) which replaces the antenna by equivalent sources on a closed surface around the antenna. The second step is to solve for the electric and magnetic fields outside of the surface surrounding the antenna. The solution to this problem is not straight-forward and is either done by an integral with a complex integrand called a Green's function\(^10\) as presented in Fig. 3.10 as the upper path, or by introducing auxiliary vector potentials and differentiating them to find the solution to the fields as indicated by the lower path in Fig. 3.10. This work makes use of the first approach to calculate the radiated fields of the slot antenna shown in Fig. 3.8. This approach boils down to a spatial convolution of the equivalent sources with their respective dyadic impulse response functions\(^11\) written as

$$E(r) = \iiint_{Q'} \tilde{G}^{e}_{\delta}(r, r') \mathbf{J}_{eq}(r') \, dr' + \iiint_{Q'} \tilde{G}^{m}_{\delta}(r, r') \mathbf{M}_{eq}(r') \, dr'.$$  \(3.42\)

In (3.42) \(r' \in Q'\) represents the spatial position of the equivalent sources on the closed surface surrounding the antenna while \(\tilde{G}^{e}_{\delta}(r, r')\) and \(\tilde{G}^{m}_{\delta}(r, r')\) denote the Green's functions for the electric field at position \(r\) due to electric and magnetic sources at position \(r'\), respectively. The result of the integral in (3.42) describes the electric field \(E(r)\) created by the antenna in terms of spatial coordinates. The expressions for the combined reactive and radiative electric field near the antenna are usually quite complicated. However, due to the rapid decay of the reactive fields only the radiative components are of interest for most practical applications\(^12\). Therefore, the electric field is usually described in the Fraunhofer region in the compact and generally valid form as

$$E(r) \big|_{r \to \infty} = E_{FF}(r) = E_0(\theta, \phi) \frac{e^{-jkr}}{4\pi r} \hat{p}_E$$  \(3.43\)

where \(E_0\) denotes the field strength of the radiated electrical field in terms of directional coordinates, \(k\) represents the wavenumber, and \(\hat{p}_E\) is a unit vector giving the

\(^9\)Also called the Huygen's Principle.
\(^10\)A Green's function is the impulse response of an inhomogeneous linear differential operator defined on a domain with specified initial or boundary conditions.
\(^11\)Dyadic analysis facilitates simple manipulation of vector field equations. The source of the electromagnetic field is given by electric and magnetic current densities, which are vector quantities. Small-signal EM fields satisfy linearity conditions, and thus the behavior of the fields can be described in terms of an impulse response. Since both input and output of the system are vector quantities the impulse response can be written as a dyadic quantity.
\(^12\)An exception to this statement is for example Near Field Communication (NFC)
direction of the electric field. From now on, the term $E_0$ normalized to its maximum value will be referred to as the radiation pattern.

The modeling of equivalent sources and the calculation of the spatial convolution, as well as the far field approximation for the slot antenna with length $l_{\text{slot}} = \Lambda \lambda_0$, is carried out in appendix A. The resulting electric field approximated in the far-field $E_{\text{FF}}$ region is expressed in terms of (3.43) as

$$E_0 = j2kV_0 \frac{\sin\left(k_x \frac{\pi A}{2}\right)}{\sin(\pi \Lambda)(k^2 - k_x^2)} \left[\cos\left(\frac{\Lambda k_x}{k}\right) - \cos(\Lambda \pi)\right]$$  \hspace{1cm} (3.44)

$$\hat{p}_{E} = \sin(\theta) \hat{\phi}$$  \hspace{1cm} (3.45)

with

$$k_x = \sin\theta \cos\phi,$$  \hspace{1cm} (3.46)

$$k_x = \cos\theta.$$  \hspace{1cm} (3.47)

The radiation pattern described by (3.44) is depicted for slot lengths $\Lambda \in \{0.5, 1, 2.5\}$ as 3D pattern as well as the cuts in the azimuth and elevation plane in Fig. 3.11.

Both (3.44) and Fig. 3.11 represent the radiation pattern of the slot in the Fraunhofer region of the antenna, which is sufficient for most application scenarios. Various parts of the radiation patterns are referred to as lobes, which might be subclassified in main, side and grating lobes [40]. A lobe usually describes a portion of a radiation pattern bounded by regions of relatively weak radiation intensity. The desired direction of radiation, which usually has the strongest radiation, is called the main lobe. The radiation patterns of the half and full wavelength slot in Fig. 3.11 show a single main lobe in broadside direction, but the beamwidth of the full-wavelength slot is decreased giving more directive radiation. This is a general result in antenna theory: It takes a larger antenna to get a more directive beam. However, this conclusion is not always obvious, since for a slot with length $l_{\text{slot}} = 1.5\lambda_0$ two main lobes appear in the radiation pattern, which is squinted symmetrically off broadside, and this is usually not desired. All three patterns in Fig. 3.11 vary only slightly with azimuthal angle due to the small width of the slot compared to the wavelength; thus the slot is considered to be an omnidirectional antenna. The radiation pattern corresponds to a Fourier transform of the equivalent current distribution that describes the antenna. Therefore, most antennas will obtain other lobes than the main lobe called sidelobes unless the aperture of the antenna is infinite, the equivalent current distribution is a Gaussian or the antenna is small enough to only show sidelobes in the evanescent domain. These sidelobes represent radiation in undesired directions, and in most applications, it is desired to keep the Sidelobe Level (SLL) as low as possible.

**Realized gain**

The two viewpoints discussed above are connected with the so-called realized gain $\Pi_r$ of the antenna's. It gives the ratio of the antennas radiation intensity, to the radiation intensity that would be obtained if the power accepted by the antenna was radiated isotropically, while taking the losses discussed in the circuit point of view into account. The electromagnetic waves radiated by an antenna transport power and energy. The Poynting vector $\mathbf{S}$ is used to relate the fields with the radiated power.
The Pointing vector describing the average radiative power density in the far-field is

\[ S(r) = \frac{1}{2} \text{Re} \left\{ E_{FF}(r) \times H_{FF}(r) \right\} \]  

(3.48)

with \( H_{FF}(r) \) denoting the magnetic field vector. In the Fraunhofer region, electric and magnetic fields are always perpendicular and related by the wave impedance \( \eta_0 \) of the propagation medium (free space), thus resulting in

\[ S(r) = \frac{1}{2 \eta_0} \frac{|E_0|^2}{r^2} \hat{r}. \]  

(3.49)

Eq. (3.49) represents the radiated power in terms of watts per square meter and as any conservative physical quantity obeys the inverse square law. The total radiated power \( P_{\text{rad}} \) from the antenna can be computed by performing an integration of the Poynting vector over a sphere \( Q \) with radius \( r \) surrounding the antenna

\[ P_{\text{rad}} = \int_Q S(r) \cdot \hat{n} \, dQ = \int_0^\pi \int_0^{2\pi} \frac{1}{2} \frac{|E_0|^2}{\eta_0} \frac{1}{r^2} r^2 \sin \theta \, d\phi \, d\theta. \]  

(3.50)

Based on the Poynting vector, a parameter called radiation intensity can be defined as

\[ U(\theta, \phi) = r^2 |S(r)| \]  

(3.51)

which describes how much power is radiated per unit solid angle in a certain direction.

With the definition of the radiation intensity it is now possible to understand the most commonly used parameter when dealing with antennas, namely the directivity. The directivity \( D \) of an antenna is a parameter that quantifies how much power is radiated in a certain angular direction with respect to an isotropic source. For the case of an isotropic radiator the radiated power is distributed equally in all directions, thus its radiation intensity \( U_{\text{iso}} \) is equal to the total radiated power divided by the solid angle of a sphere

\[ U_{\text{iso}} = \frac{P_{\text{rad}}}{4\pi}. \]  

(3.52)

The directivity is now computed by the ratio of the antenna’s radiation intensity and the radiation intensity of an isotropic source

\[ D = \frac{U(\theta, \phi)}{U_{\text{iso}}} = \frac{4\pi U(\theta, \phi)}{P_{\text{rad}}}. \]  

(3.53)

Connecting the antenna efficiency \( \epsilon_0 \) with the directivity now results in the definition of the realized gain

\[ \Pi_r(\theta, \phi) = \epsilon_0 D(\theta, \phi), \]  

(3.54)

which combines both the equivalent transmission line circuit point of view and the free space point of view into a single parameter.
Figure 3.11: Slot radiation pattern normalized to their respective maximum in 3D representation as well as cuts through azimuth and elevation plane for a)/b) half wavelength, c)/d) full wavelength and e)/f) 2.5 wavelength slots in an infinite ground plane.
3.2.2 Radiation from Antenna Arrays

In many applications like radar, satellite communication, or radio astronomy, it is necessary to design antennas with highly directive radiation characteristics. This high directivity implies a large aperture size of the selected antenna which can be achieved by two general techniques. On the one hand, it is possible to exploit reflection and refraction mechanisms using large metal surfaces or dielectric lenses to focus radiation of antennas with low directivity. A parabolic dish antenna is a prime example of this. On the other hand, it is also possible to achieve high directivity by combining multiple antennas into a so-called antenna array which focuses the radiation of the individual antennas resulting in a narrower main lobe. The radiation characteristics of an assembly of antennas depends on the relative spatial position and the excitation of individual elements. This section first introduces a mathematical description of an antenna array’s radiation pattern called the array factor for nonuniformly spaced planar arrays and moves on to describe different approaches to influence the radiation pattern of said arrays. Furthermore, a design algorithm for spatial density tapered linear arrays based on a convex optimization approach is introduced. The section closes by investigating different MIMO array topologies and their suitability for manufacturing on PCB.

Array Factor for Nonuniformly Spaced Planar Arrays

The radiation of a single antenna in the far-field region was already introduced with (3.43). Consider now an assembly of $M$ identical radiators as depicted in Fig. 3.12 with far-field radiation $\mathbf{E}_{\text{FF,}m}(r)$ located at positions

$$r_m^\prime = r_{m,x}\hat{x} + r_{m,z}\hat{z}, \quad m = 0, \ldots, M - 1$$  \hspace{1cm} (3.55)

and allowing the individual elements to be driven with unique amplitudes $A_{0,m}$ and phase $\alpha_m$. The radiation of such an assembly of individually positioned and excited antennas $\mathbf{E}_{\text{FF,}m}(r)$ can be computed by performing a vectorial superposition of the individual fields

$$\mathbf{E}_{\text{FF,}m}(r) = \sum_{m=1}^{M} E_{\text{FF,}m}(r - r_m^\prime) = \sum_{m=1}^{M} A_{0,m} E_{0,m} e^{-j(kR_m + \alpha_m)} \frac{e^{-j(kR_m + \alpha_m)}}{R_m} \hat{p}_{E,m}$$  \hspace{1cm} (3.56)

with $(\theta_m,\phi_m)$ and $\hat{p}_{E,m}$ describing the localized directional coordinates and localized polarization unit vector for the antenna at position $r_m^\prime$ and

$$R_m = r - r_m^\prime, \quad m = 0, \ldots, M - 1.$$  \hspace{1cm} (3.57)

The sum in (3.56) is not easy to interpret. However, since only the far-field is of concern for this work, the assumption $r \gg r_m^\prime$, for all $m = 0, \ldots, M - 1$ is valid and the following approximations may be applied:

- Approximation on parallel rays.
  Since $r \gg r_m^\prime$, for all $m = 0, \ldots, M - 1$, it is valid to assume that $r \parallel R_m$.
  Allowing the substitution of global directional coordinates $(\theta,\phi)$ for the localized ones $(\theta_m,\phi_m)$, and the polarization of the single antenna $\hat{p}_E$ instead of $\hat{p}_{E,m}$ for
all \( m = 0, \ldots, M - 1 \) in (3.56) results

\[
E_{\text{FF,} \Sigma}(r) = E_{0, \delta}(\theta, \phi) \hat{p}_E \sum_{m=0}^{M-1} A_{0,m} \frac{e^{-j(k \cdot R_m + \alpha_m)}}{R_m}.
\]

(3.58)

- **Approximation on amplitude.**
  
  Since \( r \gg r'_m \), for all \( m = 0, \ldots, M - 1 \), it is valid to assume that \( r = R_m \) for all \( m = 0, \ldots, M - 1 \). This allows rewriting (3.58) to

\[
E_{\text{FF,} \Sigma}(r) = E_{0, \delta}(\theta, \phi) \hat{p}_E \frac{1}{r} \sum_{m=0}^{M-1} A_{0,m} e^{-j(k \cdot R_m + \alpha_m)}.
\]

(3.59)

- **Approximation on phase.**
  
  Since \( r \gg r'_m \), for all \( m = 0, \ldots, M - 1 \), it is valid to assume that \( k||R_m \) for all \( m = 0, \ldots, M - 1 \). Allowing \( k \cdot R_m \) to be written as \( k \cdot \left| r - r'_m \right| \). Expanding \( R_m \) to the second term delivers

\[
\left| r - r'_m \right| \approx r - r'_m \left( r'_m \cdot \hat{r} \right) = r - r'_m \cdot \hat{r}
\]

(3.60)

which further simplification of (3.59) to

\[
E_{\text{FF,} \Sigma}(r) = E_{0, \delta}(\theta, \phi) \hat{p}_E \frac{e^{-jk r}}{r} \sum_{m=0}^{M-1} A_{0,m} e^{j(k \hat{r} \cdot r'_m - \alpha_m)}.
\]

(3.61)

Further introducing the substitution

\[
\hat{r} \cdot r'_m = r_{m,x} u + r_{m,x} w
\]

(3.62)

with

\[
u = \sin(\theta)\cos(\phi)
\]

(3.63)

\[
w = \cos(\theta)
\]

(3.64)
yields
\[ E_{\text{FF}, \Sigma}(r) = E_{\text{FF}, \delta}(r) \sum_{m=0}^{M-1} A_{0,m} e^{jk(r_{m,x}u + r_{m,y}v) - \alpha_m} = E_{\text{FF}, \delta} \mathcal{A}F(\theta, \phi). \] (3.65)

Eq. (3.65) now presents a very elegant conclusion. The radiation of an array of identical radiators in the Fraunhofer region can be described by the product of the radiated field of a single antenna and a factor that depends on the position and excitation of the individual array elements. This so-called array factor \( \mathcal{A}F \) is given through
\[ \mathcal{A}F(\theta, \phi) = \sum_{m=0}^{M-1} A_{0,m} e^{jk(r_{m,x}u + r_{m,y}v) - \alpha_m} \] (3.66)

is a function of the directional coordinates \((\theta, \phi)\) and is presented in the \(uvw\)-plane with coordinate points satisfying \(u^2 + w^2 \leq 1\) being called the visible region.

The array factor is interpreted as the radiation pattern of an array of antennas, and similarly to an antenna radiation pattern various parts of it are referred to as lobes. Additionally, to the main- and sidelobes that appear in an antennas radiation pattern, in an antenna array additional lobes, called grating lobes, may appear due to the spatial aliasing effect. A grating lobe is defined as "a lobe, other than the main lobe, produced by an antenna array when the inter-element spacing is sufficiently large to permit the in-phase addition of radiated fields in more than one direction" [40]. In most applications array elements are deployed on a regular lattice which is the most straightforward way since it is convenient to manufacture, simplifies the mathematical analysis and allow more efficient signal processing based on FFT algorithms. However, the simplicity of uniformly deployed antennas across the array aperture also introduces disadvantages like the appearance of grating lobes in the visible region for vast distances between array elements. Introducing some irregularity to the uniform array allows increasing the mean interelement distance to exceed half a wavelength while mitigating the effect of spatial aliasing.

**Shaping of the Array Factor**

An antenna array can be interpreted as the approximation of a continuous current distribution across the array aperture by discrete elements described with the array factor. The result obtained in (3.65) implies that the array factor can be shaped by manipulating either the excitation amplitude, phase, or the spatial position of individual elements. This manipulation is usually referred to as applying a taper function. The usual approach to applying a taper function to the array factor is to adjust the amplitude of the individual equally spaced elements \(A_{0,m}\) to approximate a current distribution such that a predefined far-field behavior is achieved. Prominent examples for amplitude tapering approaches are the Dolph-Chebyscheff taper [42], which aims to achieve the narrowest main lobe while reducing all sidelobes to a given constant level, or the Taylor taper [43] which approximates the Dolph-Chebyshev constant sidelobe level for a given number of sidelobes. A detailed overview of different amplitude taper functions is given in [44]. To realize such amplitude tapers a precise fine-tuning of the power delivered to the antenna is necessary. Similar to the amplitude tapering approach, the spatial density taper tries to approximate a continuous current distribution over the array aperture, which will yield a predefined
far-field behavior. While an amplitude taper tunes the excitation weight in order to match the desired current distribution, a density taper approach finds locations for antennas such that the element density in the aperture approximates the desired current distribution in the array aperture. The density taper approach yields the advantage that all elements are excited with the same weight, which enables all the amplifiers to be driven in equal and efficient working conditions and maximum power to be radiated [16]. Deterministic approaches for spatial density tapering exist for linear and planar arrays alike [17],[45],[46]. The deterministic spatial density tapering approach, introduced in [18], is applied and further compared to the application of an amplitude taper to a linear array with 10 elements in Fig. 3.13 for a Chebyshev taper with the goal of $-20\,\text{dB SLL}$. While Fig. 3.13(a) compares the desired Chebyshev

![Figure 3.13: Comparision of amplitude tapered (at) vs. spatial density tapered (sdt) arrays to a uniform linear array. a.) Approximated current distribution. b.) Element positions and amplitudes. c.) Resulting array factors.](image)

current distribution to an uniform one, Fig. 3.13(b) shows the result of the applied taper approaches on the excitation and position of the array elements respectively. Fig. 3.13(c) compares the resulting array factors for the uniform, amplitude tapered and spatial density tapered arrays respectively. One is able to see that both amplitude and spatial density tapers achieve the desired goal of $-20\,\text{dB SLL}$ However, the spatial density tapered array shows increasing SLL at azimuth angle approaching 0 and 180 degree. Furthermore, the spatial density tapered approach delivers a slightly increased 3 dB beamwidth compared to the amplitude taper approach. This trade-off is bought with an increase in total radiated power since all elements are driven.
uniformly in contrast to the amplitude tapered array.

Deterministic density tapering approaches like the one used in the example above lack necessary degrees of freedom, like taking into account a minimum interelement distance or multiple beam-steering angles, to be considered a sufficient design algorithm. These issues can be circumvented by resorting to design algorithms that use optimization in an iterative manner to find element positions such that given far-field requirements are met. Prior to further investigation of the design of spatial density tapered arrays, different MIMO array topologies are investigated in the proceeding section.

3.2.3 MIMO Array Topologies

In Section 3.1.4 it was already explained how the employment of the MIMO principle in terms of multiple transmit antennas with orthogonal waveforms leads to the synthesis of virtual antennas. In Section 3.2.2, the array factor was introduced as a tool to describe the far-field radiation pattern of antenna arrays. For a MIMO radar system the tow-way radiation pattern $AF_v$, that describes the radiation of the synthesized virtual array is obtained through

$$AF_v (\theta, \phi) = AF_{tx} (\theta, \phi) AF_{rx} (\theta, \phi).$$

(3.67)

Since the array factors are obtained through a Fourier transform of the element positions and the two way pattern is obtained through a multiplication of the RX and TX array factors, it can be concluded that the positions of the virtual antenna arrays are given by a spatial convolution of the element positions in the receive and transmit array respectively [47]. The MIMO principle is used in a variety of applications like near field imaging [48], [49], [50], [51], surface measurement of bulk solids [47], [52] and of course automotive radar [53]. Depending on the application and the spatial resolution is required to be two dimensional (2D) or three dimensional (3D) the employed antenna array is either linear or planar. Most MIMO array topologies aim to achieve the maximum equivalent array aperture in order to minimize the beamwidth and thus gain maximum spatial resolution as it is shown in Fig. 3.14(a). Another approach is to create the transmit and receive array such that both obtain equal beamwidths and therefore mutually supressing their respective sidelobes as it is depicted in Fig. 3.14(b) which effectively sacrifices angular resolution for sidelobe suppression. The most commonly employed planar MIMO structures like the Mills-Cross array, the T-shaped array result in a virtual array that resembles a uniform rectangular virtual array. Such arrays have the advantage that their element positions follow easy design rules. However, these designs suffer from relatively high shadowing, which results in strong sidelobes along the orthogonal planes of the array. Designs like the curvilinear [51] or logarithmic spiral array as well as designs that place elements along a fractal curve [54], aim to reduce the shadowing of elements in the virtual array and thus to spread out the sidelobe energy and to effectively decrease the SLL. Other approaches try to obtain an 'optimal' array by iteratively adapting an initial array configuration to minimize or maximize some cost function. However, such designs are time-consuming, and the 'optimality' obviously depends on the cost function involved during the optimization.

\footnote{Antenna element causes 'shadow' in the localized area behind the antenna in propagation direction. Intuitively one can imagine that the energy in the impinging wave cannot be the same if some energy is delivered to the antennas terminal.}
This work aims to employ a planar MIMO array based on SIW technology on a single layer RF substrate. The employment of a single layer severely impacts the potential complexity of the array topology since each element has to be fed by an individual waveguide which disqualifies all of the above mentioned planar topologies. In order to get a 2D topology that is easily fed on a single-layer substrate, it was decided to design for a linear array and introducing an offset orthogonal to the design dimension. This approach allows to easily feed the array elements while enabling the mapping of targets onto a 3D coordinate grid. The linear transmit and receive arrays are designed along the $x$ axis such that their sidelobes mutually suppress each other and the $-40$ dB SLL suppression requirement is fulfilled. The element positions in the linear arrays are found with the optimization algorithm introduced below and later individual TX antennas are shifted along the $z$ axis to create a planar array.

### 3.2.4 Spatial Density Tapering by Convex Optimization

Deterministic algorithms for array design based on spatial density tapering lack the option to consider system parameters like minimum element distance and multiple scanning angles in the design of the array. Due to this insufficiencies, most algorithms are not suited for the design of the later used antenna arrays. To overcome this issue, the design is done by a slightly changed version of an iterative convex optimization algorithm for linear arrays which was introduced by Yanki Aslan in [20]. The idea of the algorithm is to initially start at a uniformly distributed array and alter the inter-element distances such that the sidelobe level in the defined FOV becomes minimal. The benefit of this approach is that the formulation of the problem is done in a way, such that the minimum interelement distance and also multiple scanning angles are considered during the optimization. In the following, the formulation of the convex optimization problem is introduced.

Assume a linear array consisting of $M$ identical radiators along the $x$ axis with interelement distance $d_{\text{int}}$, for which the array factor is

$$ AF(\phi) = \sum_{m=1}^{M} e^{j\kappa]\cos(\phi)r_{m,x}}. $$

(3.68)
The \(i\)th iteration of the algorithm changes the individual elements positions to
\[
 r_{m,x}^{(i)} = r_{m,x}^{(i-1)} + \epsilon_m^{(i)}. \tag{3.69}
\]
Assuming \(\epsilon_m^{(i)}\) to be small, thus \(\nu = \left| k\cos\phi\epsilon_m^{(i)} \right| \ll 1 \rightarrow e^{j\nu} \approx 1 + j\nu\), the far field at the \(i\)th iteration can be linearized around the element position and is written as
\[
 AF^{(i)}(\phi) \approx \sum_{m=1}^{M} e^{jk\cos(\phi)r_{m,x}^{(i-1)}} \left( 1 + jk\cos\phi\epsilon_m^{(i)} \right). \tag{3.70}
\]
The optimization shall include \(p\) different scanning angles with \(\tilde{\phi}_{n} \), \(n = 1, \ldots, p\) being the direction of maximum radiation. The resulting phase shift of the \(m\)th element for scan angle \(\tilde{\phi}_{n}\), at the \(i\)th iteration is
\[
 \alpha_{m,n}^{(i)} = e^{jk\cos(\tilde{\phi}_n)r_{m,x}^{(i)}}. \tag{3.71}
\]
Substituting (3.69) and (3.71) into (3.70) and again performing a linearization with respect to the element positions results in
\[
 AF^{(i,n)}(\phi) \approx \sum_{m=1}^{M} e^{jk(\cos\phi\cos\tilde{\phi}_n)r_{m,x}^{(i-1)}} \left( 1 + jk\cos\phi\epsilon_m^{(i)} \right) \left( 1 + jk\cos\tilde{\phi}_n\epsilon_m^{(i)} \right). \tag{3.72}
\]
By neglecting the higher order terms in (3.72) the expression simplifies to
\[
 AF^{(i,n)}(\phi) \approx \sum_{m=1}^{M} e^{jk(\cos\phi\cos\tilde{\phi}_n)x_m^{(i-1)}} \left[ 1 + jk \left( \cos\phi - \cos\tilde{\phi}_n \right) \epsilon_m^{(i)} \right]. \tag{3.73}
\]
Introducing the position and shift vectors
\[
 x^{(i)} = \left[ x_1^{(i)}, x_2^{(i)}, \ldots, x_M^{(i)} \right], \tag{3.74}
\]
\[
 \epsilon^{(i)} = \left[ \epsilon_1^{(i)}, \epsilon_2^{(i)}, \ldots, \epsilon_M^{(i)} \right], \tag{3.75}
\]
and for each scan angle \(\tilde{\phi}_n\) used during the optimization a vector \(\Phi_{SL,n}\) containing the sidelobe region for each scan angle. This region is determined by a predefined beamwidth \(\phi_b^{14}\) and the angular distance between the mainlobe \(\phi_{sm}\) and the boundary of the sidelobe optimization region \(\Delta\phi_{FOV}\)
\[
 \phi \in \Phi_{SL,n}, \text{ if } \left[ \phi < \left( \tilde{\phi}_n + \Delta\phi_{FOV} \right) \text{ and } \phi > \left( \tilde{\phi}_n + \phi_b/2 \right) \right] \tag{3.76} \\
 \text{or } \left[ \phi > \left( \tilde{\phi}_n - \Delta\phi_{FOV} \right) \text{ and } \phi < \left( \tilde{\phi}_n - \phi_b/2 \right) \right]. \tag{3.77}
\]
The definition of the sidelobe region in a limited field of view is the slight change in the original algorithm introduced in [20] mentioned earlier. In order to calculate the interelement spacings during each iteration to compare them to the minimum

\(^{14}\text{Here the beamwidth between two zeros used.}\)
interelement distance \(d_{\text{min}}\) the matrix \([F]\) is formulated as

\[
[F] = \begin{bmatrix}
-1 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & \cdots & 0 & 0 & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \cdots & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & -1 & 1
\end{bmatrix}
\] (3.78)

Now the convex optimization problem to be solved is formulated as follows

\[
\min_{\epsilon(i)} \rho, \; \text{s.t.} \begin{cases}
|AF^{(i,a)}(\Phi_{\text{SL}})| \leq \rho \\
|\epsilon^{(i)}| \leq \kappa \\
[F] (\epsilon^{(i)} + x^{(i-1)}) \geq d_{\text{min}}
\end{cases}
\] (3.79)

where \(\rho\) is defined to be the maximum sidelobe level within the sidelobe regions and subject to the minimization for all scan angles \(\phi_s\) and \(\kappa\) the upper bound of position shifts during one iteration.

**Example**

The optimization algorithm to design spatial density tapered arrays is shown by example for which the input parameters given in Tab. 3.1.

<table>
<thead>
<tr>
<th>(M)</th>
<th>(d_{\text{min}})</th>
<th>(d_{\text{min}})</th>
<th>(\kappa)</th>
<th>(\phi_s ; ^{\circ})</th>
<th>(\Delta \phi_{\text{FOV}} ; ^{\circ})</th>
<th>(\phi_b ; ^{\circ})</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>0.5 (\lambda_0)</td>
<td>0.9 (\lambda_0)</td>
<td>0.08 (\lambda_0)</td>
<td>60, 90, 120</td>
<td>50</td>
<td>10</td>
</tr>
</tbody>
</table>

*Table 3.1:* Input parameters for the design example.

The resulting linear spatially density tapered array element locations are shown in Fig. 3.15(a). Furthermore the convergence of the algorithm and the resulting array factor are depicted in Fig. 3.15(b) and (c), respectively. The optimization performs well and reaches a minimum SLL of \(\rho = -24.68\) dB after approximately 30 iterations. It can be seen that the algorithm effectively optimizes the SLL within the selected sidelobe region \(\Phi_{\text{SL}}\) and ignores the resulting magnitude of sidelobes outside. For a minimum interelement distance \(d_{\text{min}} < \lambda_0/2\), grating lobes will start to appear, and for this scenario the algorithm will still perform as expected and will place the grating lobes just outside of the selected FOV. For applications where beamsteering is necessary, the FOV halves since the sidelobes (or grating lobes) will shift as much as the main lobe and will enter the FOV.

### 3.3 Slot Arrays in Waveguides

The slot antenna has already been introduced in Section 3.2.1 as a rectangular cutout in an infinite ground plane which made it easy to model the radiation characteristics but is not of much use in practical applications. A commonly used approach to employ slot antennas is to directly machine them into a waveguide that is at the same time used to feed the antenna, which significantly simplifies the design. In the following first a fundamentally excited RWG is introduced and further a single longitudinal
3.3. Slot Arrays in Waveguides

Figure 3.15: Spatial density tapered array designed with a convex optimization approach. a) Resulting element positions. b) Convergence of the algorithm. c) Resulting normalized array factor.

slot in the broad wall of said waveguide is investigated. This builds the basis for the investigation of the design of an array of slots within an RWG.

3.3.1 Single Slot in a Fundamentally Excited Rectangular Waveguide

The currents along the inner wall surfaces of an RWG are proportional to the magnetic field and narrow slots cut into the walls of the waveguide may radiate depending on its position and orientation. While a slot parallel to waveguide wall currents does not radiate a narrow slot that disrupts the flow of the wall currents, forcing them to go around the slot, will couple energy out of the waveguide through the opening into free space. Detailed discussions of the RWG and derivation of the supported TE and TM modes are commonly found in literature and are therefore not repeated here. Assuming the waveguide shown in Fig. 3.16 is excited with a Transverse Electric (TE)10 mode and terminated at its ends with a matched impedance, the dominant electric and magnetic fields in the waveguide are given by [39]

\[
E_{\text{rwg},y} = E_{\text{rwg},0} \sin(k_{c,\text{rwg}}x) e^{-j\beta_{\text{rwg}}z} \\
H_{\text{rwg},x} = -\frac{\beta_{\text{rwg}}}{\omega\mu_0} E_{\text{rwg},0} \sin(k_{c,\text{rwg}}x) e^{-j\beta_{\text{rwg}}z} \\
H_{\text{rwg},z} = \frac{j\beta_{\text{rwg}}}{\omega\mu_0} E_{\text{rwg},0} \cos(k_{c,\text{rwg}}x) e^{-j\beta_{\text{rwg}}z}
\]
with
\[ k_{c,\text{rwg}} = \frac{\pi}{a_{\text{RWG}}} \]  
representing the cut-off wavenumber, and
\[ \beta_{\text{rwg}} = \sqrt{k^2 - k_{c,\text{rwg}}^2} = \frac{2\pi}{\lambda_{\text{rwg}}} \]  
denoting the propagation constant with \( \lambda_{\text{rwg}} \) representing the corresponding wavelength. The currents in the waveguide walls can be calculated by
\[ J_{\text{rwg}} = \hat{n} \times H_{\text{rwg}} \]  
and are written in the top inner wall \( y = b_{\text{RWG}} \) as
\[ J_{\text{rwg},x} = -j \frac{k_{c,\text{rwg}}}{\omega \mu_0} E_{\text{rwg},0} \cos (k_{c,\text{rwg}} x) e^{-j \beta_{\text{rwg}} z} \]  
\[ J_{\text{rwg},x} = -\frac{\beta_{\text{rwg}}}{\omega \mu_0} E_{\text{rwg},0} \sin (k_{c,\text{rwg}} x) e^{-j \beta_{\text{rwg}} z}. \]

A longitudinal narrow slot along the z-axis in the top wall of the waveguide as depicted in Fig. 3.16(b) now disrupts the flow of the currents x-component and therefore couples energy into free space. A.F. Stevenson fist discussed the theory of slots in waveguides in [14] and concluded that the expressions for the scattered fields due to a slot in one of the walls of the waveguide are quite complicated. However, the equivalent impedance or admittance of a resonant slot in a transmission line problem exists in closed-form expressions. Stevenson’s investigation in [14] is very general as he describes slots with an arbitrary inclination in both the broad and narrow wall of the waveguide. However, in this work only the longitudinal slot in the broad wall of a RWG with an equivalent transmission line model depicted in 3.16(c) is considered.

**Slot Field**

In [15] it is found that an RWG excited with a TE\(_{10}\) mode the field in the slot aperture can be described by
\[ E_A = \frac{V_\zeta}{w_{\text{slot}}} \hat{z} \text{rect} (x - o, w_{\text{slot}}) \text{rect} (\zeta, l_{\text{slot}}) \delta (y) \]  
with \( V_\zeta \) describing the voltage distribution along the longitudinal dimension of the slot. For a slot with length \( l_{\text{slot}} \approx \lambda_{\text{eqWG}}/2 \), the dominant component of \( V_\zeta \) is a symmetrical standing wave in the form of [15]
\[ V_\zeta = V_m \sin \left( k_{\text{WG}} \left( \frac{l_{\text{slot}}}{2} - |\zeta| \right) \right), \]  
with \( V_m \) denoting the amplitude of the slot voltage. Comparing the voltage distribution of the slot in a fundamentally excited waveguide in (3.89) with the voltage distribution of a slot in an infinite ground plane with length \( l_{\text{slot}} = 0.5\lambda_0 \) as derived in appendix A(A.7), results in the conclusion that both are equal for slot lengths that are uneven integer multiples of half a wavelength. Since the fields in the slot aperture of the halfwave waveguide slot and the halfwave slot in an infinite ground plane are equal their radiation patterns will be very similar for \( y > 0 \). However, due
Figure 3.16: Single longitudinal slot in the broad wall of a RWG. a.) Rectangular waveguide dimensions. b.) Dimensions of a slot with offset from the waveguide center. c.) Equivalent transmission-line model.

to the finite dimensions of the waveguide and the mitigated radiation for \( y < 0 \) some truncation effects will be visible and the radiation pattern of the waveguide slot can not be considered omnidirectional anymore. The theoretical radiation pattern of the slot in an infinite ground plane and the one of a simulated waveguide slot is compared further below.

**Slot Admittance**

A longitudinal slot in a waveguide with characteristic admittance \( Y_0 \) is represented in an equivalent transmission-line model as a shunt admittance \( Y_{\text{slot}} = G_s + jB_s \) as depicted in Fig. 3.16(b). The analysis in [15] finds that the normalized admittance \( Y_{\text{slot}}/Y_0 \) of a single longitudinal slot in a waveguide can be expressed in terms of the reflection coefficient \( \Gamma_s \) as

\[
\frac{Y_{\text{slot}}}{Y_0} = \frac{G_s + jB_s}{Y_0} = \frac{-2\Gamma_s}{1 + \Gamma_s}
\]  

(3.90)

which proves especially useful during the numerical simulation of a single slot in chapter 4. For a resonant slot \( B_s = 0 \) the slot conductance can be expressed in closed form as [15]

\[
\frac{G_s}{Y_0} = \frac{2.09 a_{\text{RWG}} \lambda_{\text{rwg}} \cos^2 \left( \frac{\lambda_0 \pi}{2\lambda_{\text{rwg}}} \right) \sin^2 \left( \frac{\sigma \pi}{2 a_{\text{RWG}}} \right)}{b_{\text{RWG}} \lambda_0}
\]  

(3.91)

with \( \sigma \) denoting the offset of the slot from the center of the waveguide. The result in (3.91) states that the conductance of the resonant slot scales with the distance of the slot center to the center of the waveguide and thus providing the possibility to control the power radiated through this slot. This property proofs very useful when studying slot arrays in a waveguide. However, this result does not include mutual coupling
effects between arrays, and no proposition can be given about the slot susceptance and how it changes with waveguide dimension as well as slot offset.

**Slot Radiation Pattern**

Although the approach to calculate an antennas radiation pattern introduced in section 3.2.1 is generally valid it is arbitrary complex depending on the antennas geometry. Appendix A shows the calculation of the radiation pattern of a rectangular slot in an infinite ground plane. Doing the same for the slot in the RWG would, for example, require the incorporation of the geometric theory of diffraction due to the finite dimensions of the waveguide which aggravates the derivation significantly. Therefore, numerical methods are often used to investigate the radiation pattern of the complex antenna structures. Fig. 3.17 compares the radiation pattern of a half-wavelength slot in an infinite ground plane as derived above with the radiation pattern of a half-wavelength slot in an RWG. It can be seen that the radiation pattern of the slot

![Slot Radiation Pattern Diagram](image)

**Figure 3.17:** Radiation patterns of the slot in an infinite ground plane (a) and a slot in an RWG (b) and comparision of their respective directivity (c).

in a ground plane and the slot an RWG behave quite similar in the elevation plane with the difference that the back radiation is effectively mitigated ub tge RWG-case. The truncation effects of the waveguide become clearly visible in the azimuth plane and cannot be considered omnidirectional anymore. The directivity of the slot in a ground plane is as expected (from the similarity to the dipole) around 2.15 dBi while the directivity is effectively increased for the slot in an RWG due to the backradiation mitigation and narrowing of the radiation pattern in the azimuth plane.
3.3.2 Slot Arrays in fundamentally excited Rectangular Waveguide

Linear waveguide arrays generally fall into one of two possible categories, namely a standing and traveling wave array. In a traveling wave array, the magnitude of the wave decays towards the load as the energy is coupled out and radiated by the slots. The little remaining power at the end of the waveguide is absorbed by a matched load. The slots are spaced such that there is a progressive phase shift between slots, resulting in a beam that squints off broadside and scans with frequency. The antenna is well matched over a reasonably wide bandwidth, and the bandwidth does not degrade with increasing array length. In contrast to the traveling wave array, the waveguide for the linear resonant array is shorted, and a standing wave is excited within the RWG. The field pattern of a standing wave repeats with a periodicity of $\lambda_{eq\text{WG}}/2$ in the waveguide but are of opposite phase. Therefore, the slots are arranged in a $+/-$ pattern in order to excite them with equal phase and thus resulting in a broadside beam. Due to the resonative nature of this configuration, the bandwidth is usually lower than the one of a traveling wave antenna array and limited to a few percents. In the following, the guidelines for the design of a linear resonant array in a rectangular waveguide are investigated.

Design of a Linear Resonant Array

The design of an array of slots in a waveguide does not differ much from any other array design. Once the requirements of the array, such as gain, sidelobe level, beamwidth, bandwidth and polarization have been defined, the number of array elements and the individual excitations can be calculated. In a linear resonant array of longitudinal slots, the excitation of an individual element is controlled through its conductance which is effectively controlled by its offset to the center of the waveguide as suggested by (3.91).

The objective of the design is to have the admittance contribution of $M$ slots in the waveguide at the feed to be equal to the characteristic impedance of the waveguide. This guarantees a satisfying match at the center frequency. The impedance in a transmission line repeats itself after each half-wavelength section and if the admittances of the shunt slots are real at center frequency they just add to the admittance of the previous slots. Therefore, if consecutive slots are spaced half-wavelength apart minimal reflection is achieved if the normalized slot conductances satisfy

$$\sum_{i=1}^{M} \frac{G_i}{Y_0} = 1. \quad (3.92)$$

Furthermore, the last slot in the waveguide is spaced an integer multiple of $\lambda_{eq\text{WG}}/4$ away from the waveguide short as depicted in Fig. 3.18, which places all the slots on the maxima of the excited standing wave. Equations (3.91) and (3.92) would result in a simple design process if they were not neglecting the effect of mutual coupling between the slots. Mutual coupling describes the effect on the antenna’s operation if another antenna is operating nearby and usually reduces antenna efficiency. Design algorithms that incorporate mutual coupling utilize the infinite array approach which is numerically efficient but neglects array edge and truncation effects. Due to the mutual coupling effect each slot in the waveguide will obtain slightly different resonance frequencies as well as admittances. Therefore, each slot needs to be specially adjusted for their position in the array in order to be resonant and obtain the desired admittance. This significantly aggravates the array design.
3.4 Substrate Integrated Waveguides

As already mentioned in the introduction, a radar frontend PCB usually consists of various planar passive mmWave components such as antennas, transmission lines, power combiners as well as active components. In most cases, such mmWave circuits are manufactured with printed planar technologies such as stripline, microstrip, and coplanar waveguides. These technologies represent a good choice for manufacturing several passive components and antennas in the microwave region. They provide the advantage that they are compact, low profile, and easy to manufacture. However, printed planar technologies exhibit significant drawbacks due to radiation of the feed structures and from coupling to adjacent elements. For high-performance applications where losses are essential for the design and shielding of the structure is required, waveguide components are the preferred way to go. However, waveguide components are bulky, expensive, and time-consuming to manufacture, which prevents them from being used in highly integrated, low cost and high yield applications such as automotive radar.

A Japanese patent [55] proposed a technology to bridge the gap between printed planar transmission lines and waveguides by introducing the SIW to allow the implementation of classical RWG in planar form. The SIW is made up from a dielectric substrate sandwiched by two metallic planes, which resemble the top and bottom walls of the RWG, and two parallel rows of metallic through-hole connections that short the top and bottom wall to approximate the sidewalls of the rectangular waveguide. The SIW technology also called laminated waveguide, or parallel plate waveguide, is mostly attributed to the work of Ke Wu and his group at the École Polytechnique de Montréal, Canada. The technology fuses the compactness and easy manufacturing of printed planar transmission lines and the high-power handling as well as the high electromagnetic shielding capability of metallic waveguides. The SIW allows integrating a complete system consisting of antennas, filters, resonating cavities, and couplers into a single dielectric substrate, as shown in Fig. 3.19.

This section will describe the geometry and the principle of operation of SIWs. A method for analysis and the design process of an SIW is investigated with a focus on loss mechanisms and the optimal frequency of operation. The section closes with an investigation of transitions from SIW to single-ended and differential Microstrip
3.4. Substrate Integrated Waveguides

Figure 3.19: Substrate Integrated Circuit [56]

Figure 3.20: Geometry of a Substrate Integrated Waveguide.

technology.

3.4.1 Geometry and Principle of Operation

The SIW, as shown in Fig. 3.20(a), is manufactured in a dielectric substrate with top and bottom metallization that are connected by two parallel rows of tightly spaced through-hole connections (further also called vias) to approximate the sidewalls of an RWG. The behavior of the waveguide is very similar to an RWG and entirely defined by the longitudinal spacing of the vias $p_{\text{via}}$, the via diameter $d_{\text{via}}$, the distance between the parallel rows $a$, the height $b$ and the relative permittivity $\varepsilon_R$ of the substrate. As it is the case for the RWG the fundamental mode of the SIW is the TE10 mode, where the surface currents flow vertically in the sidewalls approximated by the conductive through-hole connections. As long as the consecutive vias are tightly spaced, the current on the vertical cylinders is only minimally perturbed and thus providing excellent isolation. The statement that the surface current on the sidewalls is dominantly flowing in a vertical direction is valid only for TEn0 modes. Therefore, all Transverse Magnetic (TM) and TEnp ($p \neq 0$) modes are significantly perturbed by the gaps in the sidewalls, and these modes are not able to propagate. Since all TM and TEnp ($p \neq 0$) modes are not able to propagate the thickness of the
dielectric substrate does not enter the characteristics of wave propagation but still plays a role when considering the conductive losses in the waveguide.

3.4.2 Analysis

Different modeling strategies with a significant difference in complexity have been developed to deal with the analysis of SIW structures such as interconnects, filters, and cavities. The used modeling strategy depends on the goal of the analysis. Usually, for simple interconnects, one is interested in the modal field patterns as well as the complex propagation constant for the individual modes. For such cases, the modeling of the SIW is most comfortable by considering an equivalent RWG where an analytical relationship is used to link the SIW geometry parameters to the width of the equivalent waveguide. For more complex structures such as filters, cavities, and discontinuities usually specially developed full-wave methods are used to derive the frequency behavior of the structure in terms of scattering parameters. Since the full-wave analysis methods would have exceeded the scope of the thesis, this part introduces the modeling of the SIW in terms of the equivalent rectangular waveguide model. The interested reader is referred to [57] for more information about full-wave modeling techniques for SIWs.

Equivalent Waveguide Model

This modeling approach is based on the apparent similarity between the SIW and an RWG, which leads to the assumption that an RWG as depicted in Fig. 3.16 exhibits identical dispersion characteristics as the SIW and thus the modal field patterns are described sufficiently accurate by (3.80), (3.81) and (3.82). Since the dispersion characteristics of an RWG are available in closed-form expressions, as derived in [39], the propagation constant of the SIW is trivial to find. Due to the inhomogeneity of the SIW cross-section in the longitudinal direction, the modal fields slightly change at different cross-sections. For this reason, a practical way to define the characteristic impedance of the SIW is the characteristic impedance of the equivalent waveguide [57]. Since the introduction of SIW, various equivalent waveguide models have been proposed. The first and most straight-forward one was proposed in [58], which derives an empirical relationship between the SIW parameters and the equivalent RWG width. Some models either propose more complex empirical models like in [59] to reduce the relative error further and increase the range of validity as done in [60] while other investigations use a semi-analytical approach like in [61]. In this work, the model introduced in [60] was employed since it delivers results with the smallest relative error. According to this empirically derived model, the width of the equivalent RWG is found according to

\[ a_{RWG} = \gamma a \]  

with

\[ \gamma = \zeta_1 \left( \frac{\zeta_2}{\zeta_3} - 1 \right) \]  

(3.93)

(3.94)

and

\[ \zeta_1 = 1.0198 + \frac{0.3465}{\frac{\nu_{via}}{\nu_{c}} - 1.0684} \]  

(3.95)

\[ \zeta_2 = -0.1183 - \frac{1.2729}{\frac{\nu_{via}}{\nu_{c}} - 1.2010} \]  

(3.96)

\[ \zeta_3 = 1.0082 - \frac{0.9163}{\frac{\nu_{via}}{\nu_{c}} - 0.2152} \]  

(3.97)
3.4. Substrate Integrated Waveguides

Equations (3.93)-(3.97) obtain an equivalent RWG with an error of below 1% as long as the via diameter and pitch fulfill

\[ d_{\text{via}} \leq \frac{\lambda_{\text{eqWG}}}{2}, \tag{3.98} \]

\[ p_{\text{via}} \leq 2d_{\text{via}}, \tag{3.99} \]

with \( \lambda_{\text{eqWG}} \) corresponding to the wavelength of the equivalent waveguide. The two limits in (3.99) are sufficient to ensure that the radiation loss is kept at a negligible level and a conventional rectangular waveguide can model the SIW.

3.4.3 Design

The design of a SIW can be carried out using the equivalent rectangular waveguide model introduced above. However, when comparing the equivalent RWG and the SIW, there is a massive difference in the loss mechanisms and the operational bandwidth that are not considered in the equivalent RWG model. A wave traveling in an RWG experiences dissipation of power into heat due to finite conductivity and the loss tangent of the media inside. However, a wave propagating in a SIW additionally experiences a loss due to leakage and possible bandgap effects because of the periodicity of the sidewall approximation with through-hole connections. The operational band of a SIW is, on the one hand, limited through the cut off frequency, which is accurately modeled by the equivalent RWG model. However, since the gap size of two consecutive through-hole connections gets more significant in terms of wavelength as frequency increases, it is evident that the additional loss due to leakage will introduce an upper limit on the operational band of the SIW. Furthermore, this implies an optimal frequency of operation for a given mode. In the following, the loss mechanisms and the design flow for SIW are investigated in more detail.

Loss Mechanisms

Three different mechanisms are identified that are responsible for the losses in SIW structures [62], namely conductive loss, dielectric loss, and radiation leakage, which means that the attenuation constant is written as a summation of the three main contributors

\[ \alpha_{\text{tot}} = \alpha_{d} + \alpha_{r} + \alpha_{c} \tag{3.100} \]

With \( \alpha_{d} \) accounting for the losses due to finite conductivity, \( \alpha_{d} \) for the loss due to the loss tangent of the dielectric substrate and \( \alpha_{r} \) for the attenuation due to leakage. Due to the similarity of the field propagation in RWG and SIW, the attenuation constants \( \alpha_{c} \) and \( \alpha_{d} \) for the fundamental mode of a SIW can be calculated quite accurately by using the analytical expressions for the RWG with the width of the equivalent waveguide [63]. The conductor loss

\[ \alpha_{c} = \frac{\sqrt{\pi f \epsilon_0 \sigma R}}{b \sqrt{\sigma}} \left( 1 + 2 \left( \frac{t}{f} \right)^2 \frac{b}{a_{\text{RWG}}} \right) \frac{b}{a_{\text{RWG}}} \sqrt{1 - \left( \frac{t}{f} \right)^2} \tag{3.101} \]

is approximately inversely proportional to the substrate height, and increasing the height is an effective way to reduce conductive losses in the waveguide. In contrast,
the dielectric loss

\[
\alpha_d = \frac{\pi f \sqrt{\varepsilon_R}}{c_0 \sqrt{1 - \left(\frac{d_e}{l_f}\right)^2}} \tan(\delta) \tag{3.102}
\]

is unaffected by the geometrical dimensions of the SIW. Concerning frequency behavior, it can be seen that \(\alpha_d\) is in a reasonable approximation proportional to the frequency of operation while \(\alpha_c\) is proportional to the square root of the frequency of operation. Therefore, dielectric losses become more significant for high frequencies.

The radiation leakage is dependent on the gap size between two consecutive through-hole connections, which can be mitigated by tightly spacing the vias. The lower limit is usually given by the manufacturing process, and as a rule of thumb, it is said that the leakage is negligible if \(p_{via}/d_{via} < 2.5\). Since no similar loss mechanism exists in an RWG, there is no analytical approach to evaluate these losses. For this reason, a formula has been developed for the calculation of the attenuation due to leakage in [64]

\[
\alpha_c = \frac{1}{\alpha} \left(\frac{d_{via}}{a}\right)^{2.84} \left(\frac{p_{via}}{d_{via}}\right)^{6.28} 4.85\sqrt{\left(\frac{2a}{\lambda_{c,wc}}\right)^2 - 1} \tag{3.103}
\]

The attenuation constants of the SIW as a function of the most important design parameters, namely frequency, substrate height and via spacing are depicted in Fig. 3.21. In Fig. 3.21 the frequency behaviour of the attenuation constant due to conductor, dielectric and radiation losses is depicted for \(p_{via}/d_{via} = 2\) and substrate height \(b\) equal to the thickness of the RO3003 substrate \(b_{sub}\) as defined in Tab. 2.1. Both \(\alpha_c\) and \(\alpha_d\) are highest for frequencies near cut-off while dropping to a minimum near \(f/f_c \approx 1.5\). As mentioned above the dielectric loss becomes more significant with rising frequency. Fig. 3.21(b) relates the individual attenuation constants to the substrate thickness and, as concluded above, the attenuation due to conductive losses worsens for very thin substrate. Fig. 3.21(c) shows the dependence of the radiation loss on the ratio between via diameter and via spacing of the SIW.

**Optimal frequency of operation**

The cut-off frequency and the leakage losses limit the operational band from roughly \(f_c\) to \(2f_c\). To be more precise, (3.101) - (3.103) are sufficiently accurate in the band of \(1.25f_c\) and \(1.9f_c\), and the optimal frequency of operation is

\[
f_{opt} \approx 1.4f_c. \tag{3.104}
\]

The band between \(1.25f_c\) and \(1.9f_c\) is usually called the full standard single-mode bandwidth of the SIW.
Figure 3.21: Conductive ($\alpha_c$), dielectric ($\alpha_d$) and leakage ($\alpha_r$) loss in a SIW as function of a.) excitation to cut-off frequency ($b = b_{\text{sub}} = 127\,\mu m$, $\frac{b_{\text{via}}}{d_{\text{via}}} = 2$), b.) substrate height ($f = 1.4f_c$, $\frac{b_{\text{via}}}{d_{\text{via}}} = 2$) and c.) ratio between via pitch and diameter ($f = 1.4f_c$, $b = b_{\text{sub}}$).
Chapter 4

System Design

The concepts introduced in chapter 3 are now used to explain the details and the thought process during the design of the antenna frontend. The design was carried out for a design frequency of $f_d = 77$ GHz. However, the design of the antenna element was also carried out for other frequencies to verify the design process. This chapter first investigates the construction of the SIW via the equivalent waveguide model and the calculation of its characteristic parameters. Furthermore, the dimensions of the transitions between the single-ended and differential microstrip lines to the SIW are given. The second part deals with the details of the antenna element. The first two parts were carried out with the software CST Microwave Studio which is a powerful tool for the numerical, 3D analysis of high frequency components. All design steps heavily relied on the built in automated optimization routines which allowed to adapt component parameters according to defined cost functions. The third part describes how the positions of the antenna elements in the TX and RX array have been found with the convex design algorithm introduced in Section 3.2.4. The fourth and last part of this chapter gives insight into the antenna frontend as a whole.

4.1 Substrate Integrated Waveguide Design

4.1.1 Waveguide

The parameters that influence the design of the SIW are the substrate height $h$, the metallized through hole diameter $d_{\text{via}}$, the distance between two consecutive metallized through holes $p_{\text{via}}$, as well as the distance between the parallel rows of vias $a$. The height of the substrate is already given as mentioned in chapter 2 in Tab. 2.1. Furthermore, the PCB manufacturer limited the available ratios between $d_{\text{via}}$ and $p_{\text{via}}$ and it was decided to choose a ratio that promised best manufacturing quality. Therefore, the via diameter was selected to be $d_{\text{via}} = 200 \mu m$ and the via pitch to be $p_{\text{via}} = 400 \mu m$ which is well in the range such that the radiation loss should not matter as can be seen in Fig. 3.21(c) and on the upper limit of the rule of thumbs defined in (3.99). The predefined substrate and the selection of via parameters for highest quality manufaction severely limits the degrees of freedom during the design. The only parameter left to choose is the width of the SIW which is selected such that it operates within the standard single mode bandwidth at the design frequency of $f_d = 77$ GHz.

The design of the SIW is now carried out by first selecting its desired cut-off frequency for the fundamental mode such that the optimal frequency, as defined in (3.104),
corresponds to the design frequency

$$f_d = f_{\text{opt}} \approx 1.4 f_{c, \text{SIW}} \rightarrow f_{c, \text{SIW}} \approx 55 \text{ GHz}.$$ \hspace{1cm} \text{(4.1)}$$

In order to arrive at a SIW with the desired cutoff frequency an equivalent RWG with width $a_{\text{RWG}}$ is defined and related to the width of the SIW. According to the equivalent waveguide model introduced in Sect. 3.4.2, both the SIW and the equivalent RWG should obtain the same cut-off frequency. Since the cut-off frequency for the fundamental mode of an RWG, filled with a media with relative permittivity $\varepsilon_R$ is

$$f_{c, \text{RWG}} = \frac{c_0}{2a_{\text{RWG}}\sqrt{\varepsilon_R}} \hspace{1cm} \text{(4.2)}$$

the width of the equivalent waveguides is calculated as

$$a_{\text{RWG}} = \frac{c_0}{2f_{c, \text{RWG}}\sqrt{\varepsilon_R}} = \frac{c_0 1.4}{2f_d\sqrt{\varepsilon_R}}.$$ \hspace{1cm} \text{(4.3)}$$

With the dielectric constant of the RO3003 substrate $\varepsilon_R = 3$ (4.3) results in an equivalent waveguide width of $a_{\text{RWG}} = 1.57 \text{ mm}$. Now all necessary parameters for the equivalent waveguide model are available and the width of the SIW can be calculated by numerically solving (3.93). The resulting numerical values of the equivalent waveguide model parameters from (3.94) - (3.97) are summarized in Tab. 4.1 for the resulting width of the SIW

$$a = a_{\text{RWG}}/\gamma = 1.709 \text{ mm}.$$ \hspace{1cm} \text{(4.4)}$$

<table>
<thead>
<tr>
<th>$\zeta_1$</th>
<th>$\zeta_2$</th>
<th>$\zeta_3$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1291</td>
<td>-0.5372</td>
<td>0.8025</td>
<td>0.9260</td>
</tr>
</tbody>
</table>

\textbf{Table 4.1: Parameters of the equivalent waveguide model.}

The simulation result in Fig. 4.1 shows the reflection $S_{11}$ and the transmission $S_{21}$ of the designed SIW waveguide with the solid black lines bounding the standard single mode bandwidth and the dashed black line referring to the predicted optimal frequency. Although the reflection might be lower at frequencies higher than the

![Figure 4.1](attachment:image.png)
predicted optimum, the waveguide would get bulkier for a lower cut-off frequency without significantly increasing its performance. According to the equivalent waveguide model, the fundamental mode at the design frequency should propagate with a wavelength of $\lambda_{eqWG} = 2.465 \text{ mm}$ and obtain a wave impedance of $Z_{RWG} = 313.7 \Omega$. To validate this proposition the fields in the SIW have been simulated and their dominant components are depicted in Fig. 4.2. The wave impedance of the SIW is the ratio of the transversal electric and magnetic field components and not constant with respect to the longitudinal dimension of the waveguide due to the inhomogeneous cross section. However, the wave impedance calculated according to the equivalent waveguide model is a good approximation of the mean wave impedance in the SIW. The magnitude of the different loss contributors as defined in (3.100) were determined by individual numerical simulations of a waveguide where the substrate and conductor have been exchangeably considered to be lossy and lossless. For the lossless case the conductor has been considered to be a PEC and for the lossy case, copper with a conductivity of $\sigma_d = 5.7 \times 10^7 \text{ S}$ was employed. The theoretical predictions and simulated values of the contributions to the overall loss of the SIW are summarized in Tab. 4.2. A very good agreement of the losses between the theoretically predicted

<table>
<thead>
<tr>
<th>dB cm$^{-1}$</th>
<th>$\alpha_c$</th>
<th>$\alpha_d$</th>
<th>$\alpha_l$</th>
<th>$\alpha_{tot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>theory</td>
<td>0.378</td>
<td>0.172</td>
<td>0.008</td>
<td>0.559</td>
</tr>
<tr>
<td>simulation</td>
<td>0.367</td>
<td>0.172</td>
<td>0.005</td>
<td>0.544</td>
</tr>
</tbody>
</table>

Table 4.2: Comparison of the individual loss contributors in the SIW (conductive $\alpha_c$, dielectric $\alpha_d$ and leakage $\alpha_l$) predicted by theory and obtained through simulation at the optimal frequency of operation.

values from the equivalent waveguide model and the simulated values is observed.

4.1.2 Transition from Microstrip to SIW

The SIW permits the low-cost integration of rectangular waveguides into dielectric substrates. Although the SIW got much attention over the last decade it is not commonly employed in RF systems and one of the reasons is that there are no direct
transitions from state of the art MMIC packages to the SIW available. This work employs active integrated circuits for waveform generation, power amplification and receive mixing in so-called eWLB packages and intermediate microstrip sections are necessary to connect the SIW components to these packages. The antennas in the TX array are driven by the RPN7720PL power amplifiers which offer a 100 Ω differential microstrip output while the RX antennas are supposed to connect to the 50 Ohm single-ended microstrip input of the RRN7745PN receiver. Therefore, both a single-ended and differential transition from microstrip to SIW needs to be designed. While the single-ended transition is simply done by a tapered microstrip segment that connects to the upper wall of the SIW [65] the differential transition is not so straight-forward and two possible solutions are investigated below.

**Single-Ended Transition**

In case of a single-ended microstrip to SIW transition, the geometry can be kept quite simple due to the similarity in the modal field patterns of the two structures. The transition is done by introducing a tapered microstrip section with dimensions \( w_{t, s} \) and \( l_{t, s} \) connected to the top wall of the SIW as introduced in [65] and sketched in Fig. 4.3. By adopting the tapered transition, it is generally possible to obtain an input reflection coefficient lower than −20 dB over the full standard single-mode bandwidth of the SIW. The single-ended transition as shown in Fig. 4.3 transits from the microstrip input of the RRN7745 receiver to the SIW. The design was carried out by using the design equations introduced in [65] as initial taper dimensions \( l_{t, sini} \) and \( w_{t, sini} \) and by further optimizing the taper parameters utilizing the automated optimization routines in CST for minimum reflection within the desired bandwidth. The initial dimension as well as the optimized ones \( l_{t, sopt} \) and \( w_{t, sopt} \) are given in Tab. 4.3 and the simulated scattering parameters of the optimized transition are depicted in Fig. 4.4.

![Tapered transition from single ended microstrip to SIW.](image)

**Table 4.3:** Initial and optimized taper parameters.

<table>
<thead>
<tr>
<th>( l_{t, s} )</th>
<th>( w_{t, s} )</th>
<th>( l_{t, s} )</th>
<th>( w_{t, s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.451 mm</td>
<td>0.562 mm</td>
<td>0.741 mm</td>
<td>0.539 mm</td>
</tr>
</tbody>
</table>

**Differential Transition**

The transition from a differential microstrip line to a SIW on a single layer substrate is not straight-forward. This work compares two possible solutions to this problem. One approach, further denoted as solution A, uses a rat-race coupler, as introduced in [66], as a balun to change the differential mode to a single-ended one before connecting to the SIW as sketched in Fig. 4.5(a). A second novel approach, further denoted
as solution B, as depicted in Fig. 4.5(b) splits the differential microstrip line into two individual single-ended ones and feeds them into a simple SIW power combiner structure after compensation of the differential phase has been applied. The proposed structures aim to transit from a differential microstrip line with impedance $Z_{\text{MSL,d}}$ into a SIW with impedance $Z_{\text{SIW}}$. Solution A converts the differential microstrip line

![Solution A and Solution B](image)

**Figure 4.5:** Single layer transitions from differential microstrip to SIW with a.) a rat-race coupler utilized as a balun and b.) a SIW power combiner after phase compensation.

with impedance $Z_{\text{MSL,d}} = 100 \, \Omega$, as offered by the RPN7720 power amplifier output, into a single-ended one with impedance $Z_{\text{MSL}} = 50 \, \Omega$ through a modified rat-race coupler before transitioning into the SIW. The rat-race coupler is a well-understood and commonly used device for microwave power transfer. However, this structure suffers from radiation loss and coupling to adjacent elements. Solution B describes a novel approach and first splits the differential MSL line into two independent single-ended microstrip lines, L1 and L2, by introducing a tapered transition and increasing the distance between the differential lines. The phase difference between L1 and L2 gets compensated by a 90-degree bend where radius and distance between the single-ended MSL lines are chosen such that L2 introduces an additional phase shift of $\pi$ with respect to L1. Therefore, the difference in length between L1 and L2 should correspond to half a wavelength at the design frequency, implying that their bend
radii have to satisfy the condition
\[
\frac{\pi}{2} (r_2 - r_1) = \frac{\lambda_R}{2}
\] (4.5)

with \(\lambda_R\) denoting the wavelength in the microstrip line at the design frequency. Both L1 and L2 are fed into a simplified SIW power combiner structure via tapered microstrip transitions with dimensions \(l_{t,d}\) and \(w_{t,d}\). Usually, power combiners are inherently difficult to design since one has to consider isolation, coupling, and directivity between ports that may not be excited with equal amplitude and phase. However, since for this use case, both input signals are equal in amplitude and phase (assuming narrowband operation and negligible loss over L2 compared to L1) the design of the power-combiner simplifies, since it is effectively operating as a two-port. The position of via VM together with the taper dimensions \(w_{t,d}\) and \(l_{t,d}\) are subject to an optimization such that minimum reflection occurs at the design frequency. The resulting geometrical parameters of solution B are summarized in Tab. 4.4. A numerical simulation was carried out to compare the two solutions above and the resulting scattering parameters are depicted in Fig. 4.6. Based on the simulation results in Fig. 4.6 it can be seen that the reflections of solution B can be designed to match better at certain frequencies and is thus better suited for narrowband operations. Furthermore, the transmission is approximately 0.3 dB better at the design frequency which can be traced back to the fact that solution B performs way better in terms of unwanted radiation. While in solution A the unwanted radiation amounts to \(-10\) dB, solution B looses only \(-14\) dB with respect to the excited power at the differential MSL port.

<table>
<thead>
<tr>
<th>(l_{t,d})</th>
<th>(w_{t,s})</th>
<th>(r_1)</th>
<th>(r_2)</th>
<th>(d_v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.676 mm</td>
<td>0.512 mm</td>
<td>0.741 mm</td>
<td>1.507 mm</td>
<td>0.51 mm</td>
</tr>
</tbody>
</table>

**Table 4.4:** Parameters of the differential microstrip to SIW transition using a SIW power combiner.

**Figure 4.6:** Comparision of single layer transitions from differential microstrip to SIW scattering parameters. Solution A is drawn using solid lines and solution B using dashed lines.
4.2 Antenna Element

The antenna that is employed as single element in the RX and TX array is realized as a resonant array of slots, as introduced in Sect. 3.3, within the SIW that was designed above. The radiation pattern of the antenna element is limiting the Field of View (FOV) of the array thus it is supposed to fulfill the requirements for the azimuthal and elevation beamwidth defined in chapter 2. The azimuthal beamwidth $\Theta_a$ of a resonant array of slots as depicted in Fig. 3.18 is almost an omnidirectional one since longitudinal slots are not spaced far from each other in the $x$ dimension. However, the elevational beamwidth $\Theta_e$ depends on the number of slots $M$ employed in the waveguide and can be calculated by [40]

$$\Theta_e = 2 \left[ \frac{\pi}{2} - \arccos \left( \frac{2.782\lambda_0}{2\pi d_{ant} M} \right) \right]. \quad (4.6)$$

In (4.6), variable $d_{ant}$ is fixed since the slots in a resonant linear array need to be spaced $d_{ant} = \lambda_{eq, WG} / 2$ apart. In order to get the required elevation beamwidth of $\Theta_{e, req} = 30^\circ$ the number of slots $M$ can be found by rearranging (4.6) into

$$M = \left( \frac{2.782\lambda_0}{\cos \left( \frac{\pi - \Theta_{e, req}}{2} \right) 2\pi d_{ant}} \right) = 4. \quad (4.7)$$

The individual slots with admittance $Y_{slot i} = G_{si} + jB_{si}, \ i = 1, \ldots, M$ are required to be resonant ($B_{si} = 0$) at the design frequency and the normalized slot conductances need to satisfy

$$\sum_{i=1}^{M} \frac{G_{si}}{Y_0} = 1 \quad (4.8)$$

with $Y_0$ denoting the admittance of the equivalent rectangular waveguide. The conductance of each individual slot is now chosen according to the power that needs to be radiated. This would allow to create an amplitude taper across the linear resonant array. However, the design of the individual slots proved to be difficult for the given substrate and since there was no experience with the design of SIW at such high frequency and thin substrates it was decided to keep the design simple and excite the slots uniformly. Therefore, the desired normalized slot conductance is

$$\frac{G_{si}}{Y_0} = \frac{1}{M} = 0.25. \quad (4.9)$$

In the following the design of the antenna element is carried out for three different center frequencies $f_d \in \{76 \text{ GHz}, \ 76.5 \text{ GHz}, \ 77 \text{ GHz} \}$ which are all going to be manufactured and measured in order to verify the design process. Furthermore, the design process is repeated for antenna elements with two and six slots respectively.

4.2.1 Single Slot

Chapter 3 delivered a closed expression for the normalized conductance of a single longitudinal slot in an RWG in (3.91). This should deliver the necessary slot offset to achieve the required normalized slot conductance as determined in (4.9). However, it was concluded in [67] that the model introduced in [15] for the admittance of a longitudinal slot in an RWG is less suitable for small waveguide heights. The height of the RF substrate, upon which the antenna frontend is built, is fairly small compared to the wavelength. Through simulations it was concluded that no resonant slot with
the necessary normalized conductance for this specific design frequency and substrate thickness exists. To circumvent this problem an additional degree of freedom was introduced by adding a metalized through hole connection next to the slot with distance \( g \) from the center of the waveguide, as shown in Fig. 4.7(a). The inductive post is modeled with an additional susceptance \( B_s \) in series to the slot admittance as depicted in Fig. 4.7(b) thus impacting both real and imaginary part of the admittance at position \( \xi = 0 \) in the waveguide. The approach to match a slot with an inductive post has already been successfully employed in [68]. However, no analytical model for the impact of the via on the admittance exists. Therefore, the slot length \( l_{\text{slot}} \) and width \( w_{\text{slot}} \), the slot offset \( o \) and the distance of the via from the waveguide center \( g \) were subject to an optimization which aimed to produce the normalized admittance as defined in (4.9). The optimization was carried out by creating a parameterized model and using the automatic optimization routines in CST. The optimized slot parameters for the different design frequencies and number of slots are summarized in Tab. 4.5 and their slot admittance as function of frequency is depicted in Fig. 4.8. The simulated fields in the waveguide, for a slot with normalized conductance of 0.25

<table>
<thead>
<tr>
<th>( f_d ) GHz</th>
<th>( l_{\text{slot}} ) mm</th>
<th>( w_{\text{slot}} ) mm</th>
<th>( o ) mm</th>
<th>( g ) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>76.5</td>
<td>1.695</td>
<td>0.103</td>
<td>0.231</td>
<td>0.447</td>
</tr>
<tr>
<td>77</td>
<td>1.674</td>
<td>0.102</td>
<td>0.227</td>
<td>0.454</td>
</tr>
<tr>
<td>77.5</td>
<td>1.690</td>
<td>0.104</td>
<td>0.240</td>
<td>0.431</td>
</tr>
</tbody>
</table>

Table 4.5: Optimized parameters for resonant slots with normalized conductance \( G_s/Y_0 = 0.25 \) at frequency \( f_d \).

and design frequency \( f_d = 76.5 \) GHz, are depicted and compared to the theoretical solution in Fig. 4.9. The far-field radiation pattern and directivity of the slot antenna is presented in Fig. 4.10.
Figure 4.8: Normalized slot admittances of single slots designed for resonance and desired slot conductances $G_s/Y_0$ at design frequencies 76.5 GHz (solid), 77 GHz (dashed), 77.5 GHz (dash-dotted).
Figure 4.9: Dominant electric field components in the equivalent waveguide with a longitudinal slot in its broadwall. a.) \( y \)-component of the electric field for \( y = b_{\text{sub}} \). b.) \( x \)-component for \( y = b_{\text{sub}} \). c.) \( x \)-component in the slot for \( y = b_{\text{sub}} \) and \( x = \phi \).

Figure 4.10: Directivity of a single slot antenna in an RWG as 3D and 2D representation as well as cuts through the azimuth and elevation plane.
4.2.2 Slot Array

The individually designed slots for each design frequency in the section above are now arranged into an array of \( M = 4 \) slots within the SIW. Due to mutual coupling effects the slot admittance now also depends on the position within the array which implies that the initially found slot parameters are not sufficient to achieve a well matched antenna within the desired bandwidth. In order to compensate the influence of the mutual coupling effects and get an optimally matched antenna, the parameters of each individual slot would need to be adapted during an iterative optimization. However, since there are four degrees of freedom for each slot in the array this would result in an optimization problem that is computationally very expensive. Therefore, it was decided to trade the optimal compensation of mutual coupling effects for a optimization problem with fewer degrees of freedom. In the following the detailed design results for the antenna element with four slots and center frequency \( f_d = 76.5 \) GHz are investigated since this is the element that is actually employed in the antenna frontend. The resulting parameters for the other design frequencies and number of slots are summarized further below in Tab. 4.6.

<table>
<thead>
<tr>
<th>( M )</th>
<th>( f_d ) GHz</th>
<th>( l_{\text{seq}} ) mm</th>
<th>( l_{\text{short}} ) mm</th>
<th>( l_{\text{slot}} ) mm</th>
<th>( w_{\text{slot}} ) mm</th>
<th>( o ) mm</th>
<th>( g ) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>76.5</td>
<td>2.499</td>
<td>1.541</td>
<td>1.806</td>
<td>0.101</td>
<td>0.244</td>
<td>0.438</td>
</tr>
<tr>
<td>2</td>
<td>77</td>
<td>2.467</td>
<td>1.520</td>
<td>1.724</td>
<td>0.093</td>
<td>0.238</td>
<td>0.450</td>
</tr>
<tr>
<td>2</td>
<td>77.5</td>
<td>2.499</td>
<td>1.492</td>
<td>1.685</td>
<td>0.087</td>
<td>0.233</td>
<td>0.454</td>
</tr>
<tr>
<td>4</td>
<td>76.5</td>
<td>2.499</td>
<td>1.541</td>
<td>1.807</td>
<td>0.112</td>
<td>0.292</td>
<td>0.454</td>
</tr>
<tr>
<td>4</td>
<td>77</td>
<td>2.467</td>
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<td>1.651</td>
<td>0.094</td>
<td>0.322</td>
<td>0.452</td>
</tr>
<tr>
<td>4</td>
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<td>1.492</td>
<td>1.767</td>
<td>0.107</td>
<td>0.298</td>
<td>0.427</td>
</tr>
<tr>
<td>6</td>
<td>76.5</td>
<td>2.499</td>
<td>1.541</td>
<td>1.620</td>
<td>0.114</td>
<td>0.247</td>
<td>0.316</td>
</tr>
<tr>
<td>6</td>
<td>77</td>
<td>2.467</td>
<td>1.520</td>
<td>1.651</td>
<td>0.093</td>
<td>0.322</td>
<td>0.452</td>
</tr>
<tr>
<td>6</td>
<td>77.5</td>
<td>2.434</td>
<td>1.492</td>
<td>1.529</td>
<td>0.106</td>
<td>0.271</td>
<td>0.328</td>
</tr>
</tbody>
</table>

*Table 4.6*: Geometric parameters of a resonant array of slots in a SIW with \( M \) slots, resonant at design frequency \( f_d \).

The optimization was formulated such that minimum reflection occurs at the design frequency while keeping the parameters for all slots in the array equal and carried out in two consecutive steps. First four slots with the initial parameters for 76.5 GHz as defined in Tab. 4.5 were introduced into the SIW as sketched in Fig. 4.11. The spacing between consecutive slots was initially chosen as \( l_{\text{seq}} = \lambda_{\text{seq}} WC/2 \) and the distance from
the last slot to the end (short) of the waveguide as \( l_{\text{short}} = 3\lambda_{\text{eqWG}}/4 \). However, since the short is fabricated with a wall of metallized through-hole connections, the exact position of the short is not known. In order to get the slots positioned at the maxima of the excited standing wave the parameters \( l_{\text{eq}} \) and \( l_{\text{short}} \) are subject to the first optimization that aimed to place the slots at the maxima of the excited standing wave and achieve equal phase relationship at the design frequency. In the second step the reflection from the antenna is subject to an optimization which adapts the slot length \( l_{\text{slot}} \), slot offset \( o \) as well as the via offset \( g \) of all slots to obtain a satisfying match at the design frequency. The resulting fields in the whole antenna element as well as the magnitude and phase of the fields in the individual slots that result from the two step design procedure are depicted in Fig. 4.12. Furthermore, the resulting reflection coefficient as a function of frequency is plotted in Fig. 4.13. While Fig.

**Figure 4.12:** Dominant electric-field components a.) \( E_{\text{siw,y}} \), c.) \( E_{\text{siw,x}} \) in a SIW resonant array of slots. b.) Magnitude of electric-field components in the individual slots. d.) Phases of the dominant electric-field components in the individual slots.

4.12(a) depicts the dominant \( y \) component of the electric field in the waveguide for \( y = b_{\text{sub}} \), Fig. 4.12(c) shows the \( x \)-component of the electric field which is only significant in the slots of the antenna element. The magnitude of the individual electric field components at the slot center is depicted as a function of frequency in
4.12(b) while 4.12(d) depicts the phase of the x-component in the slot. As predicted in Sect. 3.3.1 the x-component of the electric field in the slot is the dominant one. One is able to observe that the magnitude as well as the phase of the x-component of the electric field in the slots increasingly deviates the farther the slot is from the shorted end of the waveguide. This causes the bandwidth $B_{\text{ant}}$ of slot antennas to deteriorate with increasing number of slots. Nevertheless, a good match is achieved at the design frequency as it is shown in Fig. 4.13. The radiated far field is presented in Fig. 4.14 and the characteristic antenna parameters are summarized in Tab. 4.7.

<table>
<thead>
<tr>
<th>$f_d$ GHz</th>
<th>$B_{\text{ant}}$ GHz</th>
<th>$\Theta_x$ $(^\circ)$</th>
<th>$\Theta_e$ $(^\circ)$</th>
<th>$\Pi_r$ dB</th>
<th>$\epsilon_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>76.5</td>
<td>3.05</td>
<td>91</td>
<td>31</td>
<td>9.78</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Table 4.7: Simulated antenna parameters for the antenna with four slots. $f_d$ design frequency. $B_{\text{ant}}$ antenna $-10$ dB bandwidth. $\Theta_x$ $-3$ dB beamwidth in the azimuth plane. $\Theta_e$ $-3$ dB beamwidth in the elevation plane. $\Pi_r$ realized gain. $\epsilon_0$ total antenna efficiency.

The beamwidth in the elevation plane $\Theta_e$ obtains the required $30^\circ$ and a realized gain of $\Pi_r = 9.78$ dB is reached with a total efficiency of $\epsilon_0 = 0.87$. 
Figure 4.14: a.) Radiation pattern of the antenna as 3D representation. b.)c.) Antenna directivity as 2D representation as well as cuts through the azimuth and elevation plane.

4.2.3 Interelement Coupling Compensation

After the design of the antenna element a simulation was carried out to study the coupling behaviour between two antenna elements placed on the \( x \)-axis, with distance \( d_c \) apart, as sketched in Fig. 4.15(a). It was expected that the coupling would monotonically decrease with increasing distance \( d_c \) between the antennas. The simulation result shown in Fig. 4.15(b) depicts the scattering parameters between the ports of the antennas at design frequency, as a function of antenna distance \( d_c \). The dashed lines shows the initially designed antenna element. Although the loss in the SIW due to leakage is insignificant, the energy coupled out of the waveguide is sufficient to excite standing waves in the substrate and between the antennas. These standing waves severely impact the coupling behaviour between adjacent components employed in the circuit and disturbs their performance. To mitigate this problem it was decided to introduce a second row of vias in each SIW sidewall as shown in Fig. 4.15(c). The second row of metallized through hole connections in each sidewall effectively mitigates the unexpected coupling behavior and a monotonical decrease of the interelement coupling is visible, as can be seen in Fig. 4.15(b) by the solid lines. However, due to this second row of vias, the antenna element becomes bulkier and the minimum antenna element distance that needs to be considered in the next step during the design of the array topology is

\[
d_{\text{min}} = a + p_{\text{via}} \left( 1 + \sqrt{3} \right) = 2.802 \text{ mm} = 0.72 \lambda_0. \tag{4.10}
\]
Figure 4.15: Coupling between two adjacent SIW antenna elements. a.) Simulation scenario b.) Simulated reflection and transmission at design frequency for a single (dashed) and double (solid) SIW wall c.) Geometry of second wall.

Based on this investigation it was decided to outfit all SIW components, not only the antennas, with a second row of vias in their sidewalls.
4.3 Array Topology

As discussed in Sect. 3.2.3 commonly employed MIMO array topologies are difficult to manufacture within a single layer of RF substrate due to the resulting complexity of the feeding network. Therefore, it was decided to design a linear MIMO array along the z-axis that satisfies the requirements in the azimuth plane and then introduce an offset along the z-axis for individual elements resulting in a planar MIMO topology. Therefore, the design of the array topology is split into two steps. First the position of the antenna elements along the z-axis is found by employing the design algorithm for spatially density tapered arrays, based on convex optimization, as investigated in Sect. 3.2.4. Secondly to select individual elements that will obtain an offset along the z-axis in order to extend the array to a planar one will be selected.

4.3.1 Design of the Linear Arrays

The design requirement of a -40 dB SLL supression within the FOV, as discussed in Sect. 2.1.2, forced the design of the RX and TX array to have equal beamwidth which implies they mutually suppress their respective sidelobes. The input parameters for the design algorithm introduced in Sect. 3.2.4 are the number of array elements $M$, the distance between elements in the initially considered uniform array $d_{ini}$, the minimal interelement distance $d_{min}$, the upper bound for element shifts during one iteration $\kappa$, the scan angles $\phi_s$, the field of view and the width of the mainlobe. The input parameters for the design of the RX and TX array are listed in Tab. 4.8 respectively. The convergence of the iterative design algorithm for the RX and TX array is shown in Fig 4.16. The resulting element positions are depicted in Fig 4.16(b). Furthermore, the radiation patterns of the resulting TX and RX as well as the resulting virtual array are depicted in Fig. 4.17 as a function of the azimuthal angle.

As mentioned in chapter 2 the minimum achievable SLL depends on the number of employed elements. Therefore, the RX array achieves a lower sidelobe suppression within the FOV. Furthermore, the sidelobes outside the FOV are way

<table>
<thead>
<tr>
<th>M</th>
<th>$d_{ini}$</th>
<th>$d_{min}$</th>
<th>$\kappa$</th>
<th>$\phi_s$ in °</th>
<th>$\Delta \phi_{FOV}$ in °</th>
<th>$\phi_h$ in °</th>
</tr>
</thead>
<tbody>
<tr>
<td>RX</td>
<td>16</td>
<td>$d_{min}$</td>
<td>0.72$\lambda_0$</td>
<td>0.08$\lambda_0$</td>
<td>60, 90, 120</td>
<td>30</td>
</tr>
<tr>
<td>TX</td>
<td>8</td>
<td>2$d_{min}$</td>
<td>0.72$\lambda_0$</td>
<td>0.08$\lambda_0$</td>
<td>60, 90, 120</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 4.8: Input parameters to the convex optimization design algorithm for the linear TX and RX arrays.

Figure 4.16: a.) Convergence of design algorithm. b.) Resulting element positions for the RX and TX linear arrays respectively.
lower in the RX array compared to the ones in the TX array. It can be seen in Fig. 4.17 that the design goal of \(-40\) dB SLL in the MIMO two-way pattern has been reached in the desired FOV in the azimuth plane. Outside of the selected FOV grating lobes appear.

4.3.2 Extension to planar topology

For the extension to a planar topology four elements in the TX array were chosen and given an offset of \(\pm d_s\) along the \(z\)-axis and thus forming a linear array with three elements. The offset was chosen such that the beamwidth in the elevation plane becomes minimal while allowing no grating lobes within the FOV. For a linear array along the \(z\)-axis the first grating lobe appears at the elevation angle [40]

\[
\theta_{GL} = \cos^{-1} \left( \pm \frac{\lambda_0}{d_s} \right). \tag{4.11}
\]

It is desired that the grating lobe appears at the null of the element pattern (49°, from Fig. 4.14(c)) when the beam of the TX array is steered to the edge of the elevation FOV. Therefore, for broadside radiation the angle at which the grating lobe should appear is

\[
\theta_{GL} = 49^\circ - \Theta_{e,\text{req}}/2 = 34^\circ. \tag{4.12}
\]

Solving (4.11) for the element distance results in

\[
d_s = \frac{\lambda_0}{\cos (\theta_{GL})} = 1.206\lambda_0 = 4.73\text{ mm}. \tag{4.13}
\]

The resulting array element positions of the TX and RX array are depicted Fig. 4.18(a) while the positions of the virtual array elements are shown in Fig. 4.18(b). The array factor of the RX, TX and virtual array calculated according to (3.66) and is depicted in Fig. 4.19(a), (b) and (c) respectively as function of the azimuth and elevation angle. The array factor is also depicted in terms of one dimensional cuts through the azimuth and elevation planes of the RX, TX and virtual array factors in Fig. 4.19(d), (e) and (f) respectively In order to get the full picture of the array radiation one also has to consider the element pattern of the antenna element that is employed in the RX and TX array. In this case the antenna element is the same in both TX and RX array and is depicted in Fig. 4.14. The radiation of the whole array
Figure 4.18: a.) Resulting element positions for the RX and TX arrays respectively. b.) Virtual array element positions given by the convolution of RX and TX element positions.

is represented separately for the RX, TX and virtual array as function of azimuth and elevation angle as well as cuts through the azimuth and elevation plane in Fig. 4.20.
Figure 4.19: Resulting array factors in 2D representation as well as cuts through the azimuth and elevation plane. a.) b.) TX. c.) d.) RX. e.) f.) Virtual.
Figure 4.20: Resulting array radiation patterns in 2D representation as well as cuts through the azimuth and elevation plane.
   a.) b.) TX. c.) d.) RX. e.) f.) Virtual.
4.4 Antenna Frontend

At this point of the design process the antenna positions as well as the geometry of the antenna are know and now care needs to be taken to create an appropriate feeding network. The Microstrip in- and outputs of the receiver and transmitter are on opposite sides of their respective eWLB packages. It was decided to mitigate the influence of the microstrip lines as far as possible and thus to perform the transition from microstrip to SIW as soon as possible. Since the antenna ports are on opposite sides of the package a 90 degree turn had to be implemented for both RX and TX channels. Due to the compensation of the differential phase in the transition from differential microstrip to SIW the 90 degree bend is already taken care of for the TX feeds. However, for the RX feed a 90 degree bend in the SIW needed to be designed. In addition to the 90 degree turn some smoother line bends were needed to further direct the feed towards the designated antenna element. This was done by employing 45 degree bends. Both the 90 and 45 degree bend are investigated in this section which concludes the design chapter.

4.4.1 Substrate Integrated Waveguide - 90 degree bend

Since it is desirable to keep the feeding lines short and thus to keep the spacing between the receiver chips as small as possible it was decided to realize the 90 degree bend with a sharp corner as sketched in Fig. 4.21. In order to minimize the reflections caused by the sharp turn in the waveguide a through hole connection is placed in the outer corner of the bend and its position-parameter \( m \) is subject to an optimization which aims to minimize the reflection at the design frequency. The optimization delivered a position parameter of \( m = 1.202 \) mm and the resulting scattering parameters are depicted in Fig. 4.21. The introduction of metallized through hole connections into the SIW is commonly used for matching and precise power-flow control as for example in [68] to create a power divider that delivers output powers that correspond to a Dolph-Chebyshev distribution.

4.4.2 Substrate Integrated Waveguide - 45 degree bend

The 45 degree bend in the SIW is employed to direct the feeding waveguide to its respective antenna in the TX as well as in the RX array. This component is rather
trivial and depicted in Fig. 4.22(a). No special techniques have been employed during the design to optimize its behaviour since simulation showed its reflection to be below −20 dB well within the required frequency range as can be seen in Fig. 4.22(b).

\begin{figure}[h]
\centering
\begin{subfigure}[b]{0.35\textwidth}
\includegraphics[width=\textwidth]{figure_a.png}
\caption{SIW 45 degree bend. a.) Outline. b.) Simulation results.}
\end{subfigure}
\begin{subfigure}[b]{0.6\textwidth}
\includegraphics[width=\textwidth]{figure_b.png}
\end{subfigure}
\end{figure}
Chapter 5

Measurements

The designed system was manufactured after the initial design run, which is quite risky, considering that several novel elements had been employed. In order to localize errors in the case that the system performed not as expected, it was decided to manufacture PCBs with standalone components additionally. This chapter delivers an overview of individual measurement strategies, scenarios, and results of the test structures as well as for the radar system while outsourcing many of the obtained results into appendix B. Furthermore, it compares the measurements with simulated data and theoretical predictions where appropriate.

5.1 Overview

The manufactured PCBs split into two groups. Namely the system PCBs and the teststructure PCBs. First, the individual components on the test-structure PCBs were validated by utilizing a Vector Network Analyzer (VNA) with a wafer prober and WR12 waveguide transitions. Second, the system PCBs were equipped with the selected MMICs its proper working was verified performing beampattern and link-budget measurements in an anechoic chamber.

5.1.1 Teststructures

The test-structure boards, as the name suggests, provide structures for testing of individual SIW components that are employed within the system. This was done to be able to localize faulty design parts in the case, the system would not behave as desired. In order to individually measure all the SIW components, three different strategies were employed to connect a Vector Network Analyzer (VNA) to the component of interest:

- Wafer prober equipped with Air Coplanar Probe (ACP) probes in Ground-Signal-Ground (GSG) configuration to couple into a single-ended microstrip line on the PCB as depicted in Fig. 5.1(a).

- Standardized WR12 waveguide transition to a single-ended microstrip line on the PCB as seen in Fig. 5.1(b).

- Standardized WR12 waveguide transition to SIW without an intermediate step over a microstrip line as depicted in Fig. 5.1(c).
Figure 5.1: Means to connect a VNA to the component of interest on the teststructure PCBs. a.) Wafer prober with ACP probe in GSG configuration to single-ended microstrip. b.) WR12 waveguide to single-ended microstrip. c.) WR12 waveguide to SIW.

In addition to different means of connecting a VNA to the PCB, de-embedding structures for different reference planes were necessary since all were further explained below. The bulk of test components were measured utilizing the wafer prober since the transitions for these took up the least space. The waveguide transitions are bulky due to the large flange and were used to verify the measurements of the complex propagation constant. In hind sight, it would have been beneficial to manufacture the antenna elements with both wafer prober and waveguide transition, because measuring the antenna elements only on the wafer prober made a gain measurement impossible.

5.1.2 System

As the name suggests, the system PCBs inhabit the whole system, consisting of the RX and TX antenna arrays and their respective feeding, the transition from the SIW structures to the MMICs, passive components and the connection interface to the baseband hardware. Two different versions of the system PCB were manufactured. The difference between these two versions is characterized by the two different ways of feeding the differential output of the power amplifier into the SIW, as depicted in Fig. 5.2. The solution in Fig. 5.2(a) transforms the differential output of the power amplifier to a single-ended mode utilizing a rat-race coupler before transitioning into the SIW. In Fig 5.2(b), a novel structure is employed to realize the transition from the differential output of the power amplifier to the SIW. The later is considered as a safe version in case the design of the SIW power combiner would somehow fail. The system that employs the novel structure performed as intended, and the later presented results were all recorded with the system employing the novel transition from differential microstrip to SIW. The measurements were carried out in an anechoic chamber with the radar mounted on a turntable. Although the chamber was not suited ideally for measurements at such high frequency, the results are sufficient to validate the simulated beampattern of the individual TX antennas and to perform sufficient calibration measurements.
Figure 5.2: System PCB with novel transition from differential microstrip to SIW.

Figure 5.3: Transitions from a.) single-ended and b.) differential microstrip to SIW on the system PCB with equipped MMICs.
5.1.3 Applications

Different measurement scenarios with pedestrians, cyclists, and a car were recorded on a parking lot, as depicted and sketched in Fig. 5.4(a) and (b), respectively, to demonstrate the system’s capabilities concerning real-world measurements. Two scenarios are investigated in more detail:

- **Scenario 1.** Two groups consisting of two pedestrians each are moving towards each other on the sidewalk. Upon meeting each other, one member of each group stops while the respective other keeps on moving. Meanwhile, two cyclists appear on the street and in the radar’s FOV. One cyclist moves towards the radar and disappears after passing it and the other appears from behind the radar, performs a 180 degree turn and drives back.

- **Scenario 2.** A car drives on the street towards the radar while decelerating to a near stand still before accelerating into a right-turn out of the radars FOV.

Individual moving targets in theses scenarios were tracked with the help of Thomas Wagner and his algorithms for automotive radar target recognition and tracking as introduced in [69].

5.2 Components

The components manufactured on the test structure boards were measured by a Vector Network Analyzer (VNA) with both a wafer prober station and with waveguide transitions. The probing measurements were done on a Summit 12000 M Prober Station that consists of a movable chuck and two probe holders to precisely position the probes on the Device Under Test (DUT). The probing signals from the VNA are fed
into the DUT via single-mode Air Coplanar Probe (ACP) probes in Ground-Signal-Ground (GSG) configuration where the tips are spaced 150 µm apart. These probes are manually placed on special landing pads on the PCB, which allows a high-quality transition of the signal into the DUT as seen in Fig. 5.1(a). The VNA is operated with two active ports, where each port first connects to a ZVA-Z110EP extender, which upconverts the probing signal to the desired measurement frequency range of 70 GHz to 85 GHz, before transitioning to the DUT. The VNA was warmed up for 24 hours before calibration and calibrated with Through-Reflect-Line (TRL) structures on an impedance standard substrate for the prober measurements and with WR10⁴ cal kit for the waveguide measurements. This section presents the measurement results for the SIW propagation characteristics, microstrip to SIW transitions, and the antenna elements with four slots. The measurement results of de-embedding structures, SIW bends, and antenna elements with two and six slots as well as for the power combiner are presented in appendix B.

5.2.1 SIW propagation constant and wave impedance

The complex propagation constant as defined in [39] is

\[ \gamma = \alpha + j\beta \]  \hspace{1cm} (5.1)

where \( \alpha \) is the attenuation constant and \( \beta \) is the phase constant. Both the attenuation and phase constant, as well as the wave impedance of the SIW and microstrip, have been extracted from the measurements of straight transmission lines and are compared with theoretically obtained values in Fig. 5.5. The attenuation constant in a SIW has been investigated theoretically in chapter 3 as well as validated through simulations in chapter 4. Fig. 5.5 now depicts the measured attenuation constant in the SIW as a function of frequency and compares it with the theoretical prediction from the equivalent waveguide model. Although measurement and theory are off by about 0.15 dB cm⁻¹, the decreasing trend with frequency predicted by the theory is visible. It can be seen that the microstrip outperforms the SIW in terms of losses, and the measurement compares with the theoretical prediction very good. The measured phase constants for both microstrip and SIW are slightly higher than predicted by theory which directly impacts the resulting wave impedance in the SIW as seen in Fig. 5.5(c) since the wave impedance of the TE mode is inversely proportional to the propagation constant in the waveguide.

5.2.2 Transition Microstrip to Substrate Integrated Waveguide

Both the transition from single-ended and differential microstrip to SIW have been measured with the test structures depicted in Fig. 5.6(a) and (b) respectively. The transitions for both single-ended and differential microstrip have been manufactured back-to-back. The scattering parameters of a single transition have been obtained by transforming the measurements to transfer scattering representation, performing a matrix root operation, and transforming them back into standard scattering parameters. The extracted reflection and transmission parameters of a single transition from single-ended/differential microstrip to SIW are depicted in Fig. 5.6(c) and (e), as well as 5.6(d) and (f) respectively. Both transitions perform as expected with the single-ended transition, even outperforming the simulated reflection coefficient by approximately 10 dB.

¹Due to the lack of a WR12 cal kit the VNA was calibrated using a WR10 cal kit and WR10-WR12 flanges were used to transit further to the DUT
Figure 5.5: Comparision of measurement (solid) and prediction by theory (dashed) for attenuation (a) and phase constant (b) as well as wave impedance (c) in a SIW and microstrip line.
Figure 5.6: Transition from single-ended/differential microstrip to SIW teststructures a.)/b.). Comparison between measurement (solid) and simulation (dashed) of reflection parameters c.)/d.) and transmission parameters e.)/f.).
5.2.3 Substrate Integrated Waveguide Antenna - 4 Slots

The manufactured SIW antennas with four slots and different center frequencies as discussed in chapter 4 with parameters defined in Tab. 4.6 are depicted in Fig. 5.7(a), (b) and (c) respectively. The reflection coefficient of the manufactured antennas has been measured while covering the radiating slots with an absorbing material such that echoes from the measurement equipment did not influence the measurement. The resulting $S_{11}$ parameters at reference plane P2 as well as the simulated ones, are depicted in Fig. 5.7(d) for all three design frequencies. The resonance of the manufactured antenna compares very well with the simulations and is only about 500 MHz higher than the design frequency. Furthermore, for the antennas with four slots, also the secondary resonance fits the predictions, which is not the case for the antenna elements with two and six slots. The measurement data shows that the antennas $-10 \text{dB}$ bandwidth are actually higher than the simulated ones. Detailed results for measured resonance frequency $f_{\text{res}}$ and bandwidth $B_{\text{ant}}$ are gathered in Tab. 5.1.

![Image of antenna structures and graph]

**Figure 5.7:** Substrate integrated waveguide antenna with four slots and center frequencies $f_d \in \{76.5 \text{ GHz}, 77 \text{ GHz}, 77.5 \text{ GHz}\}$. a.) - c.) Teststructures. d.) Measurement results (solid) compared to simulations (dashed).

<table>
<thead>
<tr>
<th>$f_d$ GHz</th>
<th>$f_{\text{res}}$ GHz</th>
<th>$f_o$ GHz</th>
<th>$f_{\text{up}}$ GHz</th>
<th>$B_{\text{ant}}$ GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>76.6</td>
<td>76.96</td>
<td>75.06</td>
<td>78.46</td>
<td>3.4</td>
</tr>
<tr>
<td>77</td>
<td>77.68</td>
<td>75.28</td>
<td>79.2</td>
<td>3.92</td>
</tr>
<tr>
<td>77.5</td>
<td>78.02</td>
<td>75.72</td>
<td>79.48</td>
<td>3.76</td>
</tr>
</tbody>
</table>

**Table 5.1:** Measured antenna parameters. Parameters represented based on the de-embedded scattering parameters at reference plane P2.

The discussion of measurement results for the designed SIW antennas employing two and six slots with parameters found in Tab 4.6 is outsourced into the appendix B.
5.3 System

This section presents the measurements that were carried out with the fully assembled system in operation within an anechoic chamber and their corresponding results. Beampatterns of the individual TX antennas were recorded by utilizing a turntable capable of changing its angular orientation in the azimuth as well as the elevation plane. The absorbers that cover walls and objects in the anechoic chamber are not suited for such high frequencies, and multipath propagation effects are impacting measurement results. However, the influence of the low-quality absorbers was not too severe, and the measurements were sufficient to validate the simulated beampatterns as well to perform suitable calibration of the system.

5.3.1 Beampatterns

Since the individual antenna element test components were manufactured to be measured only with the wafer prober station, it was not possible to measure the beampattern of an isolated antenna element. However, the embedded beampatterns of the individual TX antennas have been measured for different frequencies in co- and cross-polarization. For this purpose, the radar system was mounted on the turntable and operated in a continuous wave mode with frequency $f_t$ while performing sweeps of the angular position. A horn antenna connected via an external mixer to a spectrum analyzer was placed in front of the radar and the measured power during the sweep was recorded. The measured beampatterns for TX antennas 1 and 4 are depicted in Fig. 5.8(a)-(d) for different transmit frequencies $f_t$. Furthermore, the measured beampatterns for TX antennas 1 and 4 at $f_t = 76.5 \text{ GHz}$ are compared to a simulation in Fig. 5.8(e) and (f) respectively. The measured beampatterns for TX antenna 1 and 4 in Fig. 5.8(a)-(b) show that the received power decreases with excitation frequency. This effect is due to decreasing available output power from the RPN7720 with increasing frequency. The cross-polarization level in Fig. 5.8(c) and (d) respectively is significantly higher in the elevation plane for small angles. This observation is because for small elevation angles, the radar system was facing the floor of the measurement chamber, and thus a significant secondary propagation path to the receive antenna was created. In all beampattern measurements of individual TX antennas in Fig. 5.8 one is able to observe a ripple of approximately 3 dB. This ripple is due to the non-ideal absorbers in the measurement chamber, which are unsuited for such high frequencies and the resulting multipath propagations as well as array edge effects, that may have introduced secondary radiation sources. This effect could have been mitigated by employing a range gate over the captured time-domain data in the receiver and using additional absorbing material to cover the edges of the frontend.
Figure 5.8: Beampattern measurement of antennas TX1/TX4 in the azimuth and elevation plane for constant transmit frequencies $f_t$. a.)/b.) Co-polarization. c.)/d.) Cross-polarization. e.)/f.) Comparision of measured (solid) and simulated (dashed) beampatterns for transmit frequency $f_t = 76.5$ GHz.
5.3.2 Calibration

A set of measurements of a single static trihedral retro-reflector was recorded for target angles in both the azimuth and elevation plane from 20° to 160° in steps of 1°. During the calibration measurements, the radar system was configured to operate in FMCW mode with parameters as introduced in chapter 3 and defined in Tab. 5.2.

<table>
<thead>
<tr>
<th>$T_{e,up}$ µs</th>
<th>$T_{e,do}$ µs</th>
<th>$f_0$ GHz</th>
<th>$B_{chirp}$ GHz</th>
<th>$N_{sample}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>256</td>
<td>32</td>
<td>76</td>
<td>2</td>
<td>1024</td>
</tr>
</tbody>
</table>

**Table 5.2:** FMCW parameters used to record calibration measurements.

For a target at broadside Fig. 5.9(a) depicts the calculated range profile of some receive channels, while Fig. 5.9(b) shows the amplitude and phase distribution across the RX array while transmit antenna 1 was active. A target at broadside is expected to produce a uniform distribution for both amplitude and phase across all receive channels. The error is due to the non-equal lengths in the receive feed lines as well as differences in individual receive mixers and amplifiers. In order to obtain meaningful results during the AoA estimation, these errors need to be compensated. Both vector and matrix calibration approaches, as introduced in Section 3.1.5, were applied to the recorded set of measurements. The results for vector and matrix calibration are compared in terms of phase error between calibrated measurement and signal model in Fig. 5.10. The phase error between calibrated measurement and signal model tends to be zero for the vector calibration at the target angle $\phi_t = 90°$, which is not surprising since this is the calibration target. However, the performance quickly deteriorates for target angles off broadside and is aggravated; the further the receive antenna is from the first one since the first antenna is used as a reference. In contrast to the vector calibration, the phase error for the matrix calibrated measurements barely exceeds 2° over a wide range of target angles. A more comprehensible comparison between the two calibration approaches is given by the maximum SLL within the
Figure 5.10: Comparison of the phase error between calibrated measurement and signal model as a function of target azimuth angle $\phi_t$. a.) Vector and b.) matrix calibration scheme.

field of view ($\phi \in [75^\circ, 105^\circ]$) as a function of the target angle as depicted in Fig. 5.11. The quality of the calibration quickly deteriorates for targets off broadside and at the edges of the FOV for the vector calibration, which is due to angle-dependent mutual coupling that is not considered for calibration with single targets. This deterioration is mitigated by the employment of the matrix-based calibration procedure, which obtains a significantly more constant sidelobe level for target angles off broadside. For target angles outside the field of view, grating lobes start to enter the region $\phi \in [75^\circ, 105^\circ]$. Therefore, the resulting rise of the maximum SLL for said target angles is not due to calibration errors, but due to the way, the arrays were designed.

5.3.3 Link Budget

The link budget analysis in a radar system accounts for all the occurring gains and losses starting from the output power of the transmitter to the target, back to the receiver all the way to the terminal of the ADC where the IF signal is discretized. The received power is a function of transmit power, target distance $R_t$ and target Radar Cross Section (RCS) and can be modeled according to the radar range equation as
introduced section 3.1.2. For the calculation of the link budget of the system during a single chirp the following parameters are considered:

- Configured output power of the RPN7720 digital power amplifier $P_{\text{gen}} = 10 \text{ mW}^2$
- Power loss over the transition from single-ended and differential microstrip to SIW.
- Power loss over the feed lines to the TX and RX antenna respectively $L_{\text{siw}} = 0.65 \text{ dB cm}^{-1}$.
- Gain of the TX and RX antenna respectively according to simulation $G_{\text{tx}} = G_{\text{rx}} = 9.82 \text{ dB}$
- RCS of the employed trihedral retro-reflector with an edge length of 7 cm $RCS_{\text{r}} = 19.77 \text{ dB m}^2$
- Conversion gain of the receive mixer $G_c = 14.5 \text{ dB}^3$
- Configured gain of the ADC $G_{\text{adc}} = 10 \text{ dB}$

The received power and the resulting voltage at the ADC terminal was calculated according to the radar range equation as introduced in 3.1.2 while considering the gains and losses listed above. In order to validate the prediction, a range gate was applied to the discretized IF signal that resulted from a retroreflector placed at variable distance $R_t$ from the radar during one chirp. The measured voltage at the ADC terminal is compared to the prediction in Fig. 5.12. When comparing the measure-

![Figure 5.12: Link budget of the radar system. Prediction compared to mean measured receive power in all channels for 4 individually active TX channels.](image)

\footnote{Value according to datasheet of RPN7720 - which is not publicly available.}

\footnote{Value according to the datasheet of the RKN774PL - which is not publicly available.}
boundary is given as
\[ R_2 = \frac{2D_{\text{ant}}^2}{\lambda_0} \] (5.2)

where \( D_{\text{ant}} \) denotes the maximum antenna dimension. For a single TX antenna with a length of approximately 1 cm the Fraunhofer boundary is somewhat 4 cm away from the antenna, so no hustle there. However, the employed corner reflector with edge length of 7 cm results in a Fraunhofer boundary of approximately 6 m. Therefore, it is comprehensible that the measurements depicted in Fig. 5.12 start to match the value and trend of the prediction only above approximately 4 m.

### 5.4 Applications

The measurements were recorded and stored on a solid-state drive with a maximum writing speed of 100 MB per second, which created a bottleneck for the amount of data that could be stored. This issue is resolved by introducing a tradeoff between unambiguous Doppler-velocity \( v_{r, \text{max}} \), cycles-per-second \( N_{\text{CPS}} \), and the number of active TX antennas to be able to store the measurement data without losing frames. For the tracking application, only a single transmit antenna was considered. All measurements were done with the FMCW parameters listed in Tab. 5.3, with \( t_{\text{wait}} \) corresponding to the delay time between consecutive up-chirps, and \( N_{\text{Loop}} \) to the number of chirps during one cycle.

<table>
<thead>
<tr>
<th>( T_{c, \text{up}} ) µs</th>
<th>( T_{c, \text{do}} ) µs</th>
<th>( t_{\text{wait}} ) µs</th>
<th>( f_0 ) GHz</th>
<th>( B_{\text{chirp}} ) GHz</th>
<th>( f_s ) MHz</th>
<th>( N_{\text{sample}} )</th>
<th>( N_{\text{Loop}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>30</td>
<td>15</td>
<td>76</td>
<td>2</td>
<td>20</td>
<td>1024</td>
<td>64</td>
</tr>
</tbody>
</table>

**Table 5.3:** FMCW parameters used to record real-world measurements.

#### 5.4.1 Scenario 1 - Pedestrians

Scenario 1 deals with pedestrians and cyclists moving around within the field of view of the radar. The measurement was recorded with \( N_{\text{CPS}} = 10 \) cycles per second with four active transmit antennas, which resulted in a maximum unambiguous Doppler-velocity of \( v_{r, \text{max}} = 2.43 \, \text{ms}^{-1} \). Fig. 5.13 depicts the scenario, the resulting range-Doppler map with a color-coded angle of arrival, as well as the result of the tracking algorithm for three different time instances during the measurement.

#### Time instance 1

Fig. 5.13 (a) shows two groups counting two pedestrians each that are moving towards each other on the sidewalk. While Group 1 moves away and is closer to the stationary radar, Group 2 is farther away and is walking towards the sensor. Two cyclists are driving in the street, and while one is very close to the sensor and moving away, the distance of the other cyclist is larger and moving towards the radar. In the range-Doppler-angle spectrum seen in Fig. 5.13(b) the groups and cyclists can be identified. While Group 2 is very spread out in the range as well as in the Doppler dimension Group 1 does not show this behavior, and the same is true for both cyclists. This observation is due to a spread in the Doppler-dimension that is caused by individually
moving parts of the target\footnote{Targets with distinct moving parts cause so-called micro-Doppler signals in the echo. These signals allow classifying the type of target. For example, a walking pedestrian has a different micro-Doppler signature than a cyclist.}. A walking pedestrian's torso is moving with a different velocity than the legs or arms attached to the pedestrian and causes different Doppler frequencies. For targets farther away from the sensor, the arms and legs might fall below the detection threshold and are thus not visible in the plot. The same is true for the cyclists, and a similar argument can be made for the spread in the range-dimension since parts of the target in different range-bins might not be strong enough to cross the detection threshold. The target detection and tracking output in Fig. 5.13(c) shows individual recognized tracks for Group 1 and 2 as well as for Cyclist 1. However, Cyclist 2 is directly in front of the radar, causing many individual reflections, and the algorithm does not recognize it as a single independently moving target. Furthermore, tracks are identified for Cyclist 1 and Group 2 but not for Group 1 since it is still too far away to be continuously recognized as a trackable target.
Time instance 2

Fig. 5.13(d) shows that Cyclist 1 has left the radar systems FOV, and Cyclist 2 is about to finish his turn while Group 1 and 2 are already very close to each other. In the range-Doppler-angle spectrum shown in Fig. 5.13(e) the individual Groups and the Cyclist are distinguishable, and the output of the tracking algorithm in Fig. 5.13(f) shows individual tracks for all independently moving objects. Also clearly visible is a smooth track for Cyclist 2, since its a single independently moving target. However, the tracks for the groups of pedestrians are more spread out since they consist of two independent targets moving in the same direction.

Time instance 3

For time instance three, as depicted in Fig. 5.13(g), Cyclist 2 has left the observation scene and furthermore, one individual from each group has stopped to great each other while the respective other in the group kept on walking. The three independently moving targets can be distinguished in the range-Doppler-angle spectrum depicted in Fig. 5.13(h) where it is worth noting that the marked blue group is visible, even though they are not moving, which is due to gesticulating and slight movement during the individual’s interaction with each other. The output of the tracking algorithm shows that instead of following the individual pedestrians that are moving past each other, it continued the track of Group 2 and drew a 180-degree turn, which does not correspond to reality. To resolve this issue, more complex tracking algorithms with the support of artificial intelligence might be employed, and radar with increased angular resolution might be helpful to distinguish individual targets in a group of pedestrians.

5.4.2 Scenario 2 - Car

Scenario 2 employed a car driving towards the stationary radar while decelerating to near stand-still and then accelerating again into a right turn away from the sensor. The measurement was recorded with $N_{CPS} = 10$ cycles per second, and only one transmit antenna active. Furthermore, the delay time between consecutive up-chirps was omitted, which resulted in an increased unambiguous Doppler-velocity of 11.6 m s$^{-1}$ compared to scenario 1. Fig. 5.14 depicts this scenario for three different time instances.

Time instance 1

Fig. 5.14(a) shows the car moving towards the radar. The range-Doppler-angle spectrum depicted in Fig. 5.14(b) shows a velocity of approximately $-5$ m s$^{-1}$ of the target as it moves towards the radar. The output of the tracking algorithm in Fig. 5.14(c) shows a single target with its respective track starting at a distance of $\approx 35$ m.

Time instance 2

Fig. 5.14(d) shows the car while it is slowing down as it gets near the radar. The target in the range-Doppler-angle spectrum in Fig. 5.14(e) is spread out in the range domain since individual reflection sources on the car, which are in different range bins like the front, the frame of the windshield, and the rear of the car are now strong enough to cross the detection threshold independently. The individual sources of reflection on the car are also visible in Fig. 5.14(f) where the tracking algorithm
Figure 5.14: Transitions from a.) single-ended and b.) differential microstrip to SIW on the system PCB with equipped MMICs.

starts to recognize two different targets as well as two different target tracks for the car.

Time instance 3

Fig. 5.14(g) shows the car as it is accelerating from a near standstill into a right turn away from the radar system. Fig. 5.14(h) shows the target with a wide range of colors indicating a very distributed target since the car occupies a wide-angle range within the FOV of the radar. The spreading in the Doppler-domain has a sharp cut near the 0 m s\(^{-1}\) due to the applied moving target filter. The output of the tracking algorithm in Fig. 5.14(i) splits up into many different targets since due to the wide occupied angle-range of the car in the FOV of the radar.
Chapter 6

Conclusion and Further Work

6.1 Conclusion

This work deals with basic theory, applied concepts, and the design process of a fully functional FMCW MIMO radar system with eight transmit and sixteen receive channels. The goal was to employ Substrate Integrated Waveguide (SIW) technology to design the antennas and their respective feed structures in cooperation with the 77 GHz radar frontend MMIC chipset from Infineon. Furthermore, the research and application of a suitable sidelobe suppression technique with the specific goal of $-40$ dB SLL within the radar systems field of view was required.

Before investigating the design of the radar system, in the first part of this thesis, a solid overview of FMCW radar systems, antennas, and antenna arrays have been addressed. Furthermore, the operational principle and loss mechanisms of the SIW, as well as the basic principle of longitudinal slot antennas in the broad wall of a waveguide, have been investigated. Based on this thorough investigation, it is concluded that SIW technology is promising and can outperform the current state of the art planar transmission lines. The smooth integration of passive components such as filters, antennas, and resonators with excellent shielding and no unintended radiation are the most persuasive arguments in favor of the SIW. Furthermore, the similarity to the rectangular waveguide allows easy modeling by equivalent waveguide models. However, the application of this technology needs to be carefully considered since the resulting structures are bulkier compared to other planar technologies. Furthermore, additional measures need to be taken for compatibility with active components embedded in integrated circuits. Antennas based on SIW technology are comfortably modeled and manufactured in the form of slots in the top waveguide wall. Concerning SLL suppression techniques, it is concluded that, although usually amplitude tapers are employed to design for a specified SLL behavior, the sidelobe suppression in this work will best be addressed by the employment of non-uniform arrays. The utilization of non-uniform antenna arrays is a great way to improve radiation characteristics of antenna arrays but comes with the price of a more substantial computational burden to calculate the angular spectrum of the received radar data and a more complicated design. Deterministic algorithms for the design of non-uniform arrays were investigated but are lacking necessary degrees of freedom to incorporate parameters like minimum interelement distance into the design. This issue can be solved by resorting to algorithms based on convex optimization allow to take the bulkiness SIW antennas and their required minimum distance to other antennas into account. The possibilities to create a planar MIMO array are impeded when considering a feed network for
the individual antenna elements on a single layer of RF substrate and planar arrays capable of 2D beamforming can only be manufactured in simple topologies.

During the design, which is essentially the second part of this work, it is concluded that, although there are analytical models for resonant slots in waveguides that would allow a straight-forward design, these models are less justified for tiny waveguide heights. An additional degree of freedom through a metalized through-hole connection near the slot is necessary to arrive at a functional design. The final antenna design relied heavily on numerical simulations and optimization tools in order to obtain resonant slots with the required conductance at the desired center frequency. The coupling between two of the designed antennas has been studied, and it was concluded that although power leaking out of the waveguide is negligible from the perspective of losses, they still excite standing waves between individual antenna elements, which impedes their performance due to the increased coupling. This behavior is mitigated by the introduction of an additional row of through-hole connections for each SIW sidewall. The out- and input of the power amplifier and receive MMICs, respectively, do not support the transition into a SIW directly. The design of suitable transitions addressed the problem of transitioning from single-ended and differential microstrip lines into a SIW. A novel solution for the transition between differential microstrip and SIW was designed, which employs a compensation of the differential phase and a simple SIW power combiner structure. The transition from differential microstrip to SIW is challenging, and the novel structure investigated works as intended but also takes up much space on the substrate.

The third part of this thesis deals with the measurement of stand-alone components as well as system performance. Individual SIW components were manufactured and measured with a wafer prober to verify the design process and the predictions made by theory as well as simulations about losses and propagation behavior. Although the measured losses are a bit higher than predicted, it is concluded that the modeling of SIW, according to rectangular waveguide models, works fine, and although the measured losses were a bit higher than predicted, it delivers reliable results. The evaluation of antenna beam patterns in an anechoic chamber shows a good agreement between measurement and simulation. Furthermore, the investigation of the system performance via a link budget analysis proved the successful assembly and operation of the radar system. The comparison of calibration approaches based on phase error to a theoretical model shows the influence of mutual coupling in the measurements of single static targets. It can be concluded that calibration methods that rely on a single measurement of a single static target are outperformed by approaches where multiple targets are employed since they allow to account for angle-dependent coupling effects. Radar measurements for moving targets such as pedestrians and cyclists have been performed outside in a parking lot, which showed the real-world applicability of the system.

The design of the radar system was limited only by the two given research topics, namely the utilization of SIW and SLL suppression techniques. These two very general topics provided the possibility for extensive and exciting research. However, they also delivered many degrees of freedom concerning the actual radar system design. With hindsight, many little details catch the eye that would better be done differently or leave room for optimization. For example, the definition of the field of view in the array topology design and the extension of the linear array in order to obtain a planar array leave room a more sophisticated design.
6.2 Future Topics

The presented work covers a range of topics and shows very satisfying results, both from a simulation and measurement perspective. However, some potential topics that could further be addressed or based on this work are:

- An obvious extension to this work concerns the array topology design. While the convex optimization algorithm was only carried out for the linear array design, it is conceivable that an extension to an optimization working with planar array positions would deliver better results.

- Another possible area for improvement would be the calibration of the array. Although the two introduced approaches worked well for this system, an approach that takes angle-dependent mutual coupling and other effects into account may boost the system performance further.

- A big topic would be the improvement of SIW waveguides. With the development of empty SIW exciting innovative circuit and mmWave component designs will be possible with substrate integrated technology for frequencies up to 350 GHz.
Appendix A

Radiation pattern of the slot antenna

Consider a rectangular slot with length $l_{\text{slot}}$ and width $w_{\text{slot}}$ in an infinite groundplane fed by a co-planar waveguide as depicted in Fig. 3.8. In an ideal case the transmission line does not effect the radiation of the structure and is neglected for the further derivation of the radiation pattern. Without the waveguide feed, the slot can be represented as a planar electric field distribution as depicted in Fig. A.1.

![Electric field distribution within the slot aperture.](image)

**Figure A.1:** Electric field distribution within the slot aperture.

If the slots width is much smaller than the wavelength it is safe to assume that the electric-field distribution satisfies

$$E_A = \frac{V_{\text{slot}}(x)}{w_{\text{slot}}} \text{rect}(x, w_{\text{slot}}) \hat{x}$$  \hspace{1cm} (A.1)

with $V_{\text{slot}}$ representing the voltage distribution in the slot. First an expression for the voltage distribution and thus the field within the slot is derived by using an equivalent
transmission line model and second the radiation pattern in the far-field is calculated by the means of a dyadic impulse response function.

A.1 Field distribution in the slot

in order to find the voltage distribution, the slot is unfolded and only a shorted co-planar waveguide is used to model the antenna, resulting in the equivalent transmission line model depicted in Fig. A.2.

\[ V_{\text{slot}}(x) = V^+ e^{-jtx} + V^- e^{jtx} = V^+ \left( e^{-jtx} + \Gamma_v e^{jtx} \right) \]  \hspace{1cm} (A.2)

with \( \Gamma_v \) denoting the voltage reflection coefficient. Due to the short at the end of the transmission line the voltage needs to go to zero there

\[ V_{\text{slot}}(x = \frac{l_{\text{slot}}}{2}) = 0 = V^+ \left( e^{-jl_{\text{slot}}/2} + \Gamma_v e^{jl_{\text{slot}}/2} \right) \]  \hspace{1cm} (A.3)

and the voltage reflection coefficient reduces to

\[ \Gamma_v = e^{-jl_{\text{slot}}} \]  \hspace{1cm} (A.4)

resulting in a sinusoidal voltage distribution along the transmission line

\[ V^+ \left( e^{-jtx} + e^{-jl_{\text{slot}}} e^{jtx} \right) = V^+ e^{-\frac{jkl_{\text{slot}}}{2}} \left( e^{\frac{jkl_{\text{slot}}}{2}} e^{-jtx} + e^{-\frac{jkl_{\text{slot}}}{2}} e^{jtx} \right) \]

\[ = V^+ e^{-\frac{jkl_{\text{slot}}}{2}} 2\sin \left( k \left( \frac{l_{\text{slot}}}{2} - x \right) \right) \]  \hspace{1cm} (A.5)

Defining the voltage at the origin to be

\[ V_{\text{slot}}(0) = V_0 = V^+ e^{jl_{\text{slot}}/2} 2\sin \left( \frac{k l_{\text{slot}}}{2} \right) \]  \hspace{1cm} (A.6)

the final expression for the voltage distribution in the slot is

\[ V_{\text{slot}}(x) = V_0 \frac{\sin \left( k \left( \frac{l_{\text{slot}}}{2} - x \right) \right)}{\sin \left( \frac{k l_{\text{slot}}}{2} \right)} . \]  \hspace{1cm} (A.7)
Folding the slot back, implying a change in variable and substituting the result in A.7 into A.1 gives

\[ E_A = V_0 \sin \left( k \left( \frac{k_{\text{diag}}}{2} - |z| \right) \right) \frac{\sin \left( \frac{k_{\text{diag}}}{2} \right) \sin \left( \frac{x}{w_{\text{slot}}} \right) \text{rect} \left( \frac{x}{w_{\text{slot}}} \right)}{w_{\text{slot}}} \hat{z}. \]  

(A.8)

Folding the slot back to its original form implies that the impedance is not equal to the one of the coplanar waveguide anymore and special measures have to be taken to match the antennas input impedance to the characteristic impedance of the transmission line. However, the expression for the voltage distribution in the slot is still expected to be a very good approximation.

A.2 Equivalent Sources

Utilizing the boundary condition in (3.39) the field in the slot is expressed in terms of a surface magnetic current distribution

\[ M_s = E_A \times \hat{n} \]  

(A.9)

that is present over the entire area on the slot. Because of the boundary condition over the PEC surface with \( \hat{n} \) representing the unit vector orthogonal to the aperture. The tangential components of the electric field are zero and therefore, no magnetic but only electric currents \( J_s \) are present on the ground plane as suggested by Fig. A.3.

![Figure A.3: Electric and magnetic surface sources for the slot antenna.](image)

In order to calculate the electromagnetic field above the ground plane \( (y > 0) \), the equivalent current distribution is derived by using the equivalence theorem [41]. The half space below the ground plane is considered to be the equivalent region and for this kind of problem it is convenient to fill the equivalent region with PEC as shown in Fig. A.4

![Figure A.4: Electric and magnetic surface sources for the slot antenna with PEC equivalent region in the lower halfspace.](image)
As a consequence, the image theorem can be applied and the electric current distribution $J_s$ on the PEC surface is zero, since they have an image on the symmetry plane ($y = 0$) that are oriented in the opposite direction, effectively canceling each other as seen in Fig. A.5.

![Electric and magnetic surface sources for the slot antenna with image theorem applied.](image)

**Figure A.5:** Electric and magnetic surface sources for the slot antenna with image theorem applied.

The magnetic currents in contrary have an image oriented the same way, therefore effectively, simplifying the whole problem to an equivalent magnetic current distribution in the slot aperture equal to

$$M_{eq} = 2M_s = 2E_A \times \hat{n} \quad (A.10)$$

as depicted in Fig. A.6.

![Equivalent magnetic current distribution describing the slot antenna in an infinite ground plane.](image)

**Figure A.6:** Equivalent magnetic current distribution describing the slot antenna in an infinite ground plane.

Substituting the field in the slot aperture from (A.8) into (A.10) gives the final expression for the equivalent magnetic source

$$M_{eq} = 2V_0 \frac{\sin \left( k \left( \frac{w_{slot}}{2} - |z| \right) \right)}{\sin \left( k \frac{w_{slot}}{2} \right) w_{slot}} \text{rect} \left( \frac{x}{w_{slot}} \right) \text{rect} \left( \frac{z}{l_{slot}} \right) \delta \left( y \right) \hat{z}. \quad (A.11)$$

The problem has now been reduced to an equivalent magnetic current distribution in free space as depicted in Fig. A.7 that is further used to find the radiated electric field in the far-field.
A.3 Calculation of the radiated fields

Since the modeling of the equivalent sources was done with a PEC filled equivalent region the problem is described only in terms of equivalent magnetic sources as sketched in Fig. which simplifies (A.12) to

\[ E(r) = \int \int_{Q'} \hat{G}_0^{em}(r, r') M_{eq}(r') \, dr'. \]  \hspace{1cm} (A.12)

with

\[ \hat{G}_0^{em} = - \left( \nabla \times \hat{I} \right) G_0(r - r') = - \begin{pmatrix} 0 & -\frac{\partial}{\partial z} & -\frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & -\frac{\partial}{\partial x} \\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{pmatrix} G_0(r - r') \]  \hspace{1cm} (A.13)

and

\[ G_0(r - r') = \frac{e^{-jk|r-r'|}}{4\pi |r-r'|} \]  \hspace{1cm} (A.14)

representing the scalar Green's function.

The dyadic Green's function \( \hat{G}_0^{em}(r - r') \) represents the spatial vector impulse response in free space for the electric field at point \( r \) due to a magnetic source at point \( r' \) in Cartesian coordinates. It is common practice to simplify A.14 for calculation points \( r \gg \frac{1}{k} \) by using the radiative dyadic Green's function and thus avoiding the derivative operations. With the simplified version of the Greens function the field is expressed through

\[ E(r) = -jk \int \int_{Q'} \left( \hat{R} \times \hat{I} \right) G_0(r - r') M_{eq}(r') \, dr'. \]  \hspace{1cm} (A.15)
The integral in A.15 is still not straight-forward to solve but can be further simplified applying three approximations\(^1\) that will deliver the electric-field in the Fraunhofer region \(E_{FF}\) as an easy to solve integral:

- **Approximation on parallel rays**
  For \(r \gg r'\) it can be assumed that vectors \(\mathbf{R}\) and \(\mathbf{r}\) are parallel. Therefore it is possible to write \((\mathbf{R} \times \mathbf{I}) = (\mathbf{r} \times \mathbf{I})\) and put the expression out of the integral.

- **Approximation on amplitude**
  For \(r \gg r'\) it is valid to approximate the distance from source point \(r'\) to observation point \(r\) with the distance from origin to the observation point. Thus, \(R = r\) and allowing the denominator of the scalar Green’s function to be written outside of the integral.

- **Approximation on phase**
  Since \(r \gg r'\), it is valid to assume that \(k||\mathbf{R}\). Allowing \(k \cdot \mathbf{R}\) to be written as \(kR = k|\mathbf{r} - \mathbf{r}'|\). Expanding \(R\) to the second term delivers

\[
|\mathbf{r} - \mathbf{r}'| \approx r - r'(\mathbf{r}' \cdot \mathbf{\hat{r}}) = r - r' \cdot \mathbf{\hat{r}} \tag{A.16}
\]

which allows to further split the phase of the scalar Green’s function and put one part outside of the integral.

Applying the approximations introduced above and keeping in mind that \(\mathbf{M}_{eq}(r') = \mathbf{M}_{eq}(r')\mathbf{\hat{z}}\) allows to rewrite equation A.15 into

\[
\mathbf{E}(r) \approx \mathbf{E}_{FF}(r) = -j\mathbf{k} \left[ (\mathbf{\hat{r}} \times \mathbf{I}) \cdot \mathbf{\hat{z}} \right] \frac{e^{-jkr}}{4\pi r} \iint_{Q'} \mathbf{M}_{eq}(r')e^{jk\mathbf{r}' \cdot \mathbf{\hat{r}}'} \, dr'. \tag{A.17}
\]

Equation A.17 now presents elegant conclusion that the radiated electric-field in the Fraunhofer region is essentially given by the product of a spherical wave, the spatial Fourier transformation of the equivalent current distribution and a term describing the transverse components. Rewriting the scalar product between the wave vector and the the source point results in

\[
k \cdot \mathbf{r}' = k(\sin\theta\cos\phi\mathbf{\hat{x}} + \sin\theta\sin\phi\mathbf{\hat{y}} + \cos\theta\mathbf{\hat{z}}) \mathbf{r}' = k_x x' + k_y y' + k_z z' \tag{A.18}
\]

and allows to split the Fourier transform in equation A.17 into its cartesian components and to be written as

\[
\iint_{Q'} \mathbf{M}_{eq}(r')e^{j(k_x x' + k_y y' + k_z z')} \, dx' \, dy' \, dz'. \tag{A.19}
\]

Integrating in each spatial variable while substituting \(l_{slot} = \Lambda\lambda_0 = \frac{2\pi}{k}\) for the slot length results in

\(^1\)These are the same approximations that have been applied during the derivation of the array factor in the Fraunhofer region in chapter 2.
A.3. Calculation of the radiated fields

- **x:**

\[
\int_{-\infty}^{\infty} \text{rect} \left( x', w_{\text{slot}} \right) e^{jk_x x'} \, dx' = \int_{w_{\text{slot}}/2}^{w_{\text{slot}}/2} e^{jk_x x'} \, dx' \\
= \frac{1}{jk_x} \left( e^{jk_x w_{\text{slot}}/2} - e^{-jk_x w_{\text{slot}}/2} \right) = \frac{1}{k_x} 2\sin \left( k_x \frac{w_{\text{slot}}}{2} \right) \\
= w_{\text{slot}} \text{sinc} \left( k_x \frac{w_{\text{slot}}}{2} \right)
\]

- **y:**

\[
\int_{-\infty}^{\infty} \frac{2V_0}{w_{\text{slot}}} \delta (y') e^{jk_y y'} = \frac{2V_0}{w_{\text{slot}}}
\]

- **z:**

\[
\int_{-\infty}^{\infty} \text{rect} \left( z', l_{\text{slot}} \right) \frac{\sin \left( k \left( \frac{l_{\text{slot}}}{2} - |z'| \right) \right)}{\sin \left( k \frac{l_{\text{slot}}}{2} \right)} e^{jkz'} \, dz' = \int_{-\Lambda_z^+}^{\Lambda_z^+} \frac{\sin \left( \pi \Lambda - k |z'| \right)}{\sin \left( \pi \Lambda \right)} e^{jkz'} \, dz' \\
= \frac{1}{\sin \left( \pi \Lambda \right)} \left[ \int_{-\Lambda_z^+}^{0} \sin \left( \pi \Lambda + k z' \right) e^{jkz'} \, dz' + \int_{0}^{\Lambda_z^+} \sin \left( \pi \Lambda - k z' \right) e^{jkz'} \, dz' \right] \\
= \frac{2k \left[ \cos \left( \frac{\Lambda_z k_x}{k} \right) - \cos \left( \Lambda \pi \right) \right]}{\sin \left( \Lambda \pi \right) \left( k^2 - k_x^2 \right)}
\]

Evaluating the transverse components in spherical coordinates results in

\[
\left[ (\hat{r} \times \hat{I}) \cdot \hat{z} \right] = \left[ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right] \times \left[ \begin{array}{c} I_x \\ I_y \\ I_z \end{array} \right] = \left( \begin{array}{c} \cos \theta \\ -\sin \theta \\ 0 \end{array} \right) = \left( \begin{array}{c} 0 \\ 0 \\ -\sin \theta \end{array} \right) = -\sin \theta \phi. \tag{A.20}
\]

Putting together the individual expressions calculated above, the far-field of a slot is expressed through

\[
E_{\text{FF}} (r) = j2kV_0 \frac{e^{-jkr} \text{sinc} \left( k_x \frac{w_{\text{slot}}}{2} \right) \left[ \cos \left( \frac{\Lambda \pi k_x}{k} \right) - \cos \left( \Lambda \pi \right) \right]}{4\pi r} \sin \left( \frac{\Lambda \pi}{k^2 - k_x^2} \right) \sin (\theta) \phi. \tag{A.21}
\]
Appendix B

Measurement Results

B.1 De-embedding

In order to eliminate the impact of the transition between the ACP probe and the PCB as well as the impact of the transition between the microstrip line and the SIW, TRL structures were employed to define reference planes for scattering parameters. In the post-processing these reference planes are used to calculate the scattering parameters of the actual DUT. The manufactured de-embedding structures for both reference planes P1 and P2 in single ended transitions, as sketched in Fig. B.1(a), are depicted in Fig. B.1(b) and (c) respectively. The measured scattering parameters for the Through, Reflect and Line structures for reference planes P1 and P2 are depicted in Fig. B.2 and B.3 respectively.

The transition from the probe to a single-ended microstrip is not sufficient to test all employed components. In order to test the differential feeding structure discussed in Sect. 4.1.2 a differential input signal is necessary. Since there was no probe available that was suited for this task, a rat-race coupler was employed to transform the single-ended mode into a differential one as sketched in Fig. B.4. The measured scattering parameters of the differential de-embedding structures for the reference plane Pd are depicted in Fig. B.5.
Figure B.2: Measured scattering parameters of the de-embedding structures for the single-ended transition reference plane P1.

Figure B.3: Measured scattering parameters of the de-embed structures for the single-ended transition reference plane P2.

Figure B.4: Differential transition de-embedding structures. a.) Outline with definition of reference plane Pd. b.) Manufactured de-embedding structures for reference plane Pd.

During the design of the differential test components an unintended error was introduced into the design. The differential de-embedding structures as seen in Fig. B.4 are designed such that they are meant to be probed from the same side. This implies that the probe holder on the probing station would have to be placed next to each other. However, since the prober holders are very bulky, it was not possible to connect the structures as intended and they had to be probed from opposite sides. This implied that while one differential transition is connected properly the other is
Figure B.5: Measured scattering parameters of the de-embedding structures for the differential transition reference plane Pd.

probed such that the probe is above the rat-race coupler, which obviously impacts its performance. This fact is visible in the measurement results of the de-embedding structures in Fig. B.5 where the $S_{11}$ parameter of the Through and the Line behaves as expected and the $S_{22}$ parameter shows no sharp dips and barely goes below $-10$ dB. For the de-embedding of the measured differential transition a cheat was applied and the $S_{22}$ parameter was set equal to $S_{11}$.

The procedure to calculate the scattering parameters of the DUT was implemented according to the algorithm introduced in [70].

B.2 Substrate Integrated Waveguide Bend - double 45 degree

The test structure for the double 45 degree bend that is employed in the antenna frontend to match the $x$ position of the feed after transitioning from the MMICs to the SIW with the $x$ position of the antenna is depicted in Fig. B.6(a). The reflection and transmission scattering parameters, de-embedded for reference plane P2, are compared to the simulation results in Fig. B.6(b) and (c) respectively. Simulation
and measurement agree very well and a reflection coefficient of approximately $-20 \text{ dB}$ is achieved over a wide frequency range.

### B.3 Substrate Integrated Waveguide Bend - 90 degree

The manufactured test structure for the double 90 degree bend that is employed in the feed for the individual receive antennas is depicted in Fig. B.6 (a). The test structure was manufactured as two 90 degree bends back to back and the scattering parameters of a single bend was extracted by performing a transformation to transfer scattering parameters and performing a matrix root operation. The originally measured and extracted scattering parameters for reference plane P2 as well as the simulation result are depicted in figure B.6 (b).

![Image](image.png)

Figure B.7: Substrate integrated waveguide 90 degree bend. (a) Teststructure. (b) Measurement results.

### B.4 Antenna Elements

#### B.4.1 2 Slots

The manufactured substrate integrated waveguide antennas with six slots and different center frequencies as discussed in Chapter 4 with parameters defined in Tab. 4.6 are depicted in Fig. B.8(a), (b) and (c) respectively. The $S_{11}$ parameter of the antennas have been measured while covering the radiating slots with an absorbing material such that echoes from the measurement equipment did not influence the measurement. The measured scattering parameters, deembedded for reference plane P2, are compared to the simulated ones in Fig. B.8(d). The resonance of the manufactured antenna compares very well with the simulations and is only about 500 MHz higher than the design frequency. However, the secondary resonance that is also seen in the simulations is about 2.5 GHz lower than predicted during the simulations. The measurement data shows that the antennas $-10 \text{ dB}$ bandwidth is actually higher than the simulated one which is especially remarkable for the antenna with a design frequency of $f_d = 77.5 \text{ GHz}$ as seen in figure B.8 (d), where the first and second resonance are so close together that the reflection does not go above $-10 \text{ dB}$ between them. Detailed results for resonance frequency and bandwidth are gathered in Tab. B.1.
Figure B.8: Substrate integrated waveguide antenna with two slots and center frequencies \( f_d \in \{76.5 \text{ GHz}, 77 \text{ GHz}, 77.5 \text{ GHz}\} \). a) - c.) Teststructures. (b) Measurement results (solid) compared to simulations (dashed).

<table>
<thead>
<tr>
<th>( f_d ) GHz</th>
<th>( f_{\text{res}} ) GHz</th>
<th>( f_{\text{lo}} ) GHz</th>
<th>( f_{\text{up}} ) GHz</th>
<th>( B_{\text{ant}} ) GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>76.6</td>
<td>77.14</td>
<td>75.36</td>
<td>78.7</td>
<td>3.34</td>
</tr>
<tr>
<td>77</td>
<td>77.5</td>
<td>75.82</td>
<td>79.72</td>
<td>3.9</td>
</tr>
<tr>
<td>77.5</td>
<td>78.08</td>
<td>76.49</td>
<td>83</td>
<td>6.51</td>
</tr>
</tbody>
</table>

Table B.1: Measured parameters of SIW antenna with two slots. Parameters represented based on the de-embedded scattering parameters at reference plane P2.

### B.4.2 Substrate Integrated Waveguide Antenna - 6 Slots

The manufactured substrate integrated waveguide antennas with six slots and different center frequencies as discussed in Chapter 4 with parameters defined in Tab. 4.6 are depicted in Fig. B.9 (a), (b) and (c) respectively. The \( S_{11} \) parameter of the antennas has been measured while covering the radiating slots with an absorbing material such that echoes from the measurement equipment did not influence the measurement. The measured scattering parameters, deembedded for reference plane P2, are compared to the simulated ones in Fig. B.9(d) While the measurements and simulations of the SIW antennas with two and four slots agree reasonably well in terms of resonance frequency and bandwidth this is not the case for the antenna element with six elements. The simulations of the six slot SIW antenna delivered a resonance at the desired frequency. However, the simulations also already showed a severe decrease in bandwidth. Besides the element with design frequency \( f_d = 77.5 \text{ GHz} \) depicted in Fig. B.9(d) the design of the antennas with six slots can be considered a failure. The reason for this is hard to estimate without a further manufacturing run.
**Appendix B. Measurement Results**

**Figure B.9:** Substrate integrated waveguide antenna with six slots and center frequencies $f_d \in \{76.5 \text{ GHz} \ 77 \text{ GHz} \ 77.5 \text{ GHz}\}$. a.) - c.) Teststructures. (d) Measurement results (solid) compared to simulations (dashed).

### B.5 Beampattern
Figure B.10: Beampattern measurements for TX antenna 2/3. 
\[ \text{a.}/\text{b.) Co-polarization. c.}/\text{d.) Cross-polarization. e.}/\text{f.) Comparison of to the simulated embedded pattern at } f = 76.5 \text{ GHz.} \]
Figure B.11: Beampattern measurements for TX antenna 5/6. 
(a)/(b) Co-polarization. (c)/(d) Cross-polarization. (e)/(f) Comparison of to the simulated embedded pattern at $f = 76.5$ GHz.
Figure B.12: Beampattern measurements for TX antenna 7/8.
a.)/b.) Co-polarization. c.)/d.) Cross-polarization. e.)/f.) Comparison of to the simulated embedded pattern at $f = 76.5$ GHz.
B.6 SIW power combiner

Figure B.13: Teststructures of the SIW power combiner a.) manufactured back to back and b.) with one port matched via a microstrip spiral and their respective measured scattering parameters c.) back to back and d.) matched.
B.7 WR12 waveguide to SIW transition

Figure B.14: Transition from WR12 waveguide to SIW. a.) Test structure. b.) Comparison between simulated (dashed) and measured (solid) scattering parameters.
## Symbols

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>Free space speed of light sort</td>
</tr>
<tr>
<td>$AF$</td>
<td>Array Factor</td>
</tr>
<tr>
<td>$AF_{rx}$</td>
<td>Array Factor of the receive array</td>
</tr>
<tr>
<td>$AF_{tx}$</td>
<td>Array Factor of the transmit array</td>
</tr>
<tr>
<td>$AF_v$</td>
<td>Array Factor of the synthesized virtual array</td>
</tr>
<tr>
<td>$\alpha_{tot}$</td>
<td>Total attenuation constant in a SIW structure</td>
</tr>
<tr>
<td>$\alpha_c$</td>
<td>Attenuation constant in a SIW structure due to finite conductivity</td>
</tr>
<tr>
<td>$\alpha_d$</td>
<td>Attenuation constant in a SIW structure due to non ideal dielectric</td>
</tr>
<tr>
<td>$\alpha_t$</td>
<td>Attenuation constant in a SIW structure due to leakage effects</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Attenuation constant.</td>
</tr>
<tr>
<td>$A_{0,m}$</td>
<td>Excitation amplitude of the $m$th element in the array</td>
</tr>
<tr>
<td>$A_0$</td>
<td>Amplitude of transmit signal</td>
</tr>
<tr>
<td>$\theta_{GL}$</td>
<td>Angle of the first order grating lobe for a linear array along the z-axis.</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Incident angle of plane wave on a linear receive array measured from broadside</td>
</tr>
<tr>
<td>$\delta_{inc}$</td>
<td>Angular resolution</td>
</tr>
<tr>
<td>$a_{RWG}$</td>
<td>Width of an Rectangular Waveguide</td>
</tr>
<tr>
<td>$a$</td>
<td>Width of a substrate integrated waveguide defined between the via centers</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Ratio between the width of the SIW and its equivalent RWG</td>
</tr>
<tr>
<td>$B_{ant}$</td>
<td>Antenna bandwidth</td>
</tr>
<tr>
<td>$B_{chirp}$</td>
<td>FMCW chirp bandwidth</td>
</tr>
<tr>
<td>$\beta_{CPW}$</td>
<td>Propagation constant of a coplanar waveguide</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Phase constant</td>
</tr>
<tr>
<td>$\beta_{rwg}$</td>
<td>Phase constant in an rectangular waveguide</td>
</tr>
<tr>
<td>$b_{RWG}$</td>
<td>Height of an Rectangular Waveguide</td>
</tr>
<tr>
<td>$b$</td>
<td>Height of the substrate integrated waveguide. Corresponds to the substrate layer height</td>
</tr>
<tr>
<td>$[C]$</td>
<td>Coupling matrix</td>
</tr>
<tr>
<td>$q_e$</td>
<td>Electric charge distribution</td>
</tr>
<tr>
<td>Notation</td>
<td>Description</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$q_{e,s}$</td>
<td>Surface electric charge distribution</td>
</tr>
<tr>
<td>$q_{m,s}$</td>
<td>Surface magnetic charge distribution</td>
</tr>
<tr>
<td>$q_m$</td>
<td>Magnetic charge distribution</td>
</tr>
<tr>
<td>$J_{rwg}$</td>
<td>Current density in the sidewalls of a rectangular waveguide.</td>
</tr>
<tr>
<td>$J_{rwg,x}$</td>
<td>$x$-component of the current density in the sidewalls of a rectangular waveguide.</td>
</tr>
<tr>
<td>$J_{rwg,z}$</td>
<td>$z$-component of the current density in the sidewalls of a rectangular waveguide.</td>
</tr>
<tr>
<td>$H_{rwg}$</td>
<td>Magnetic field in an rectangular waveguide.</td>
</tr>
<tr>
<td>$J_{eq}$</td>
<td>Equivalent electric current density vector</td>
</tr>
<tr>
<td>$J$</td>
<td>Electric current density vector</td>
</tr>
<tr>
<td>$J_s$</td>
<td>Surface electric current density vector</td>
</tr>
<tr>
<td>$M$</td>
<td>Magnetic current density vector</td>
</tr>
<tr>
<td>$M_{eq}$</td>
<td>Equivalent magnetic current density vector</td>
</tr>
<tr>
<td>$M_s$</td>
<td>Surface magnetic current density vector</td>
</tr>
<tr>
<td>$D_{ant}$</td>
<td>Maximum antenna dimension</td>
</tr>
<tr>
<td>$D$</td>
<td>Directivity of an antenna</td>
</tr>
<tr>
<td>$d_c$</td>
<td>Distance between two antennas during the coupling study.</td>
</tr>
<tr>
<td>$d_{ini}$</td>
<td>Initial interelement distance for iterative design algorithm of spatial density tapered array</td>
</tr>
<tr>
<td>$d_{min}$</td>
<td>Minimum distance of antenna elements</td>
</tr>
<tr>
<td>$d_{rx}$</td>
<td>Distance of antenna elements in the receive array</td>
</tr>
<tr>
<td>$d_{ant}$</td>
<td>Distance of consecutive slots in the antenna element</td>
</tr>
<tr>
<td>$d_{tx}$</td>
<td>Distance of antenna elements in the transmit array</td>
</tr>
<tr>
<td>$d_v$</td>
<td>Distance of the matching via in Solution B for the transition from differential Microstrip to SIW.</td>
</tr>
<tr>
<td>$d_a$</td>
<td>Angle of the first order grating lobe for a linear array along the $z$-axis.</td>
</tr>
</tbody>
</table>

- Scalar product of $e$:
  
  - Euler's number
  
  - $\epsilon_{cd}$: Combined conductive and dielectric efficiency of antenna
  
  - $\epsilon_r$: Reflection (misalignment) efficiency of antenna
  
  - $\epsilon_0$: Antenna efficiency
  
  - $\vec{p}_E$: Unit vector of the electric field
  
  - $\vec{P}_{E,m}$: Unit vector of the electric field radiated from $m$th array element
  
  - $E_A$: Electric field in the slot aperture.
  
  - $E_d$: Difference of electric field vector
  
  - $E_{FF}$: Electric field vector approximated in the far-field (Fraunhofer) region
<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{r_wg,y}$</td>
<td>$y$-component of the electric field in an rectangular waveguide.</td>
</tr>
<tr>
<td>$E$</td>
<td>Electric field vector</td>
</tr>
<tr>
<td>$E_{FF,\delta}$</td>
<td>Radiated field in the Fraunhofer region from a single radiator</td>
</tr>
<tr>
<td>$E_{FF,\Sigma}(r)$</td>
<td>Radiated field in the Fraunhofer region from a single radiator</td>
</tr>
<tr>
<td>$E_{r_wg,0}$</td>
<td>Amplitude of the electric field in an rectangular waveguide.</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Element position change during one iteration</td>
</tr>
<tr>
<td>$E_0$</td>
<td>Radiation pattern of single antenna</td>
</tr>
<tr>
<td>$E_{0,\delta}$</td>
<td>Radiation pattern of $m$th element in array of $M$ elements</td>
</tr>
<tr>
<td>$\eta_0$</td>
<td>Free space wave impedance</td>
</tr>
</tbody>
</table>

$[F]$ Maximum sidelobe level within the field of view that is subject to the optimization

$f$ Frequency

$f_c$ Waveguide cut-off frequency

$f_{c,LPF}$ Cut-off frequency of the lowpass filter

$f_{beat}$ Frequency of the beat signal

$\Delta f_{beat}$ Resolution bandwidth of the beat signal

$f_{c,SIW}$ SIW design cut-off frequency

$f_d$ Design frequency

$f_{lo}$ Lower frequency bounding antenna $-10$ dB bandwidth.

$f_{doppler}$ Frequency offset due to Doppler effect

$f_{opt}$ Waveguide optimal frequency of operation

$f_{res}$ Measured resonance frequency of designed antennas.

$f_s$ Sampling frequency

$f_0$ FMCW chirp start frequency

$f_{up}$ Upper frequency bounding antenna $-10$ dB bandwidth.

$f_{c,RWG}$ RWG cut-off frequency

$f_t$ Static transmit frequency

$g$ Distance of the slot matching via from the center of the waveguide

$\Psi$ Amplitude ratio between received and transmitted signal

$G_{ade}$ Gain of the ADC.

$G_c$ Conversion gain of the receive mixer.

$\Pi_r$ Realized gain

$G_{rx}$ Realized gain of the RX antenna.

$RCS_t$ Radar-Cross-Section of the target.

$G_{tx}$ Realized gain of the TX antenna.

$\gamma$ Complex propagation constant.
<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
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<td>$\tilde{G}^{e_j}_0$</td>
<td>Dyadic impulse response function of the electric field due to an electric current density in free space</td>
</tr>
<tr>
<td>$\tilde{G}^{em}_0$</td>
<td>Dyadic impulse response function of the electric field due to a magnetic current density in free space</td>
</tr>
<tr>
<td>$\mathbf{I}$</td>
<td>Unit dyad</td>
</tr>
<tr>
<td>$G_0$</td>
<td>Scalar greens function in free space</td>
</tr>
<tr>
<td>$\infty$</td>
<td>Infinity</td>
</tr>
<tr>
<td>$j$</td>
<td>Imaginary unit</td>
</tr>
<tr>
<td>$k$</td>
<td>Norm of the wave vector</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Upper boundary for position change during one iteration</td>
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<td>$k_{c,\text{rgw}}$</td>
<td>Cut-off wavenumber in an rectangular waveguide.</td>
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<tr>
<td>$k$</td>
<td>Wave vector</td>
</tr>
<tr>
<td>$k_{\text{VG}}$</td>
<td>Norm of the wave vector in a waveguide</td>
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<td>x-component of the wave vector</td>
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<tr>
<td>$k_y$</td>
<td>y-component of the wave vector</td>
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<tr>
<td>$k_z$</td>
<td>z-component of the wave vector</td>
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<tr>
<td>$\lambda_{\text{rgw}}$</td>
<td>Wavelength in an rectangular waveguide.</td>
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<td>$L_{\text{siw}}$</td>
<td>Power loss ratio in the antenna feed lines in dB.</td>
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<td>Distance between consecutive slots in an Substrate Integrated Waveguide (SIW) slot array.</td>
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<tr>
<td>$l_{\text{short}}$</td>
<td>Distance from the center of the last slot to the end of the waveguide in an Substrate Integrated Waveguide (SIW) slot array.</td>
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<td>$l_{\text{slot}}$</td>
<td>Slot length</td>
</tr>
<tr>
<td>$l_{t,d}$</td>
<td>Length of the taper for differential transition from Microstrip to SIW</td>
</tr>
<tr>
<td>$l_{t,s}$</td>
<td>Length of the taper for single-ended transition from Microstrip to SIW</td>
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<tr>
<td>$H_d$</td>
<td>Difference of magnetic field vector</td>
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<tr>
<td>$H_{\text{FF}}$</td>
<td>Magnetic field vector approximated in the far-field (Fraunhofer) region</td>
</tr>
<tr>
<td>$H_{\text{rgw},x}$</td>
<td>x-component of the magnetic field in an rectangular waveguide.</td>
</tr>
<tr>
<td>$H_{\text{rgw},z}$</td>
<td>z-component of the magnetic field in an rectangular waveguide.</td>
</tr>
<tr>
<td>$\nabla$</td>
<td>Nabla operator</td>
</tr>
<tr>
<td>$\mathbf{n}$</td>
<td>Surface unit vector</td>
</tr>
<tr>
<td>Notation</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
</tr>
<tr>
<td>$M$</td>
<td>Number of array elements</td>
</tr>
<tr>
<td>$N_p$</td>
<td>Number of utilized measurements for least squares problem to find transformation matrix</td>
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<tr>
<td>$N_{rx}$</td>
<td>Number of receive antennas</td>
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<tr>
<td>$N_{sample}$</td>
<td>Number of samples recorded during a single chirp.</td>
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<tr>
<td>$N_{tx}$</td>
<td>Number of transmit antennas</td>
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<tr>
<td>$N_v$</td>
<td>Number of synthesized virtual antennas</td>
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<tr>
<td>$o$</td>
<td>Offset of the longitudinal slot from the center of the waveguide</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
</tr>
<tr>
<td>$\partial$</td>
<td>Symbolizing partial derivative</td>
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<tr>
<td>$c$</td>
<td>Complex-valued coupling coefficient between antenna elements</td>
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<tr>
<td>$\mu_0$</td>
<td>Permeability of free space</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>Permittivity of free space</td>
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<tr>
<td>$\varepsilon_R$</td>
<td>Dielectric constant for an isotropic and homogeneous dielectric</td>
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<tr>
<td>$\Delta \alpha_{rx}$</td>
<td>Phase difference between two RX channels due a plane wave with oblique incident on the receive array</td>
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<td>$\alpha_m$</td>
<td>Excitation phase of the $m$th element in the array</td>
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<td>$\alpha_{rx}$</td>
<td>Phase of the receive signal</td>
</tr>
<tr>
<td>$\alpha_{tx}$</td>
<td>Phase of the transmit signal</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Azimuthal angle $\phi$ for the spherical coordinate system definition</td>
</tr>
<tr>
<td>$\hat{\phi}$</td>
<td>Scan angle</td>
</tr>
<tr>
<td>$\phi_t$</td>
<td>Target azimuthal angle.</td>
</tr>
<tr>
<td>$\hat{\phi}$</td>
<td>Unit vector for azimuthal angle $\phi$ in the spherical coordinate system definition</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Mathematical constant equal to a circle’s circumference divided by its diameter</td>
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<tr>
<td>$P_{gen}$</td>
<td>Power generated at the output of the RPN7720 digital power amplifier.</td>
</tr>
<tr>
<td>$P_{rad}$</td>
<td>Total radiated power</td>
</tr>
<tr>
<td>$P_{rx}$</td>
<td>Received power.</td>
</tr>
<tr>
<td>$P_{tx}$</td>
<td>Transmitted power.</td>
</tr>
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<td>$z$-Coordinate of the Cartesian coordinate system</td>
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<tr>
<td>$\hat{z}$</td>
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# Acronyms

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<td>2D</td>
<td>two dimensional</td>
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<tr>
<td>3D</td>
<td>three dimensional</td>
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<tr>
<td>ACP</td>
<td>Air Coplanar Probe</td>
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<tr>
<td>ADAS</td>
<td>Advanced Driver Assistance Systems</td>
</tr>
<tr>
<td>ADC</td>
<td>Analog to Digital Converter</td>
</tr>
<tr>
<td>AoA</td>
<td>Angle of Arrival</td>
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<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
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<tr>
<td>CMOS</td>
<td>Complementary Metal-Oxide-Semiconductor</td>
</tr>
<tr>
<td>CPW</td>
<td>Coplanar Waveguide</td>
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<tr>
<td>CST</td>
<td>Computer Simulation Technology</td>
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<tr>
<td>CW</td>
<td>Contious Wave</td>
</tr>
<tr>
<td>DUT</td>
<td>Device Under Test</td>
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<tr>
<td>EM</td>
<td>Electromagnetic</td>
</tr>
<tr>
<td>eWLB</td>
<td>Embedded Wafer Level Ball Grid Array</td>
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<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>FMCW</td>
<td>Frequency Modulated Continuous Wave</td>
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<tr>
<td>FOV</td>
<td>Field of View</td>
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<tr>
<td>FPGA</td>
<td>Field Programmable Gate Array</td>
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<tr>
<td>GSG</td>
<td>Ground-Signal-Ground</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronic Engineers</td>
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<tr>
<td>IF</td>
<td>Intermediate Frequency</td>
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<tr>
<td>ITU</td>
<td>International Telecommunication Union</td>
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<tr>
<td>LIDAR</td>
<td>Light Detection and Ranging</td>
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<tr>
<td>LNA</td>
<td>Low Noise Amplifier</td>
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<tr>
<td>LO</td>
<td>Local Oscillator</td>
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<tr>
<td>MIMO</td>
<td>Multiple Input Multiple Output</td>
</tr>
<tr>
<td>MMIC</td>
<td>Monolithic Microwave Integrated Circuit</td>
</tr>
<tr>
<td>MSPS</td>
<td>Mega Samples per Second</td>
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<tr>
<td>NFC</td>
<td>Near Field Communication</td>
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<tr>
<td>PA</td>
<td>Power Amplifier</td>
</tr>
<tr>
<td>PCB</td>
<td>Printed Circuit Board</td>
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<tr>
<td>PEC</td>
<td>Perfect Electric Conductor</td>
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<tr>
<td>PLL</td>
<td>Phase Locked Loop</td>
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<tr>
<td>PMCW</td>
<td>Phase Modulated Continuous Wave</td>
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<tr>
<td>PRI</td>
<td>Pulse Repetition Interval</td>
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<tr>
<td>RADAR</td>
<td>Radio Detection and Ranging</td>
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<tr>
<td>RCS</td>
<td>Radar Cross Section</td>
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<tr>
<td>RF</td>
<td>Radio Frequency</td>
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<tr>
<td>RWG</td>
<td>Rectangular Waveguide</td>
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<tr>
<td>RX</td>
<td>Receiver</td>
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<tr>
<td>SIMO</td>
<td>Single Input Multiple Output</td>
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<tr>
<td>SIW</td>
<td>Substrate Integrated Waveguide</td>
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<td>SLL</td>
<td>Sidelobe Level</td>
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<td>TDMA</td>
<td>Time Division Multiple Access</td>
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<td>TE</td>
<td>Transverse Electric</td>
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<tr>
<td>TM</td>
<td>Transverse Magnetic</td>
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<td>TQFP</td>
<td>Quad Flat Package</td>
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<td>TRL</td>
<td>Through-Reflect-Line</td>
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<td>TX</td>
<td>Transmitter</td>
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<td>US</td>
<td>United States</td>
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<tr>
<td>USB</td>
<td>Universal Serial Bus</td>
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<tr>
<td>VCO</td>
<td>Voltage Controlled Oscillator</td>
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<td>VNA</td>
<td>Vector Network Analyzer</td>
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Bibliography


[2] C. Hulsmeyer, Hertzian wave projection and receiving apparatus adapted to indicate or give warning of the presence of a metallic body, such as a ship or a train, in the line of projection of such waves, U.K. Patent 13170, 1904.


