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Novel and efficient simulation approach for effective permeabilities of randomly ordered two-phase compounds

Neuer und effizienter Simulationsansatz für effektive Permeabilitäten von zufällig geordneten und zweiphasigen Materialmischungen

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Abstract: This paper introduces a novel simulation approach for the magnetic properties of two-phase randomly ordered compounds. In industry, materials such as ferrous powder mixtures or metallic granulates are very often used as raw materials. Hence, their material characteristics are of utmost interest for material manufacturers in order to guarantee high quality standards. Typically, many parameters such as composition, inclusion shape, and the characteristics of the constituents affect the macroscopic physical behavior of such materials. In particular, the resulting permeability of multi-phase and randomly ordered materials exhibits a strong variation despite constant compounds. For the design and optimization of measurement setups, efficient simulators are necessary to estimate the effective permeability and its fluctuation range of a huge number of arrangements. In addition to the basic concept of the novel simulation method, this article presents some possible evaluations of the simulated results and their dependencies on the properties of the constituents. In the last century, a large number of different mixing formulas have been established in literature, which are summarized and compared to the simulation results. Finally, the simulated magnetic characteristics are evaluated with finite element simulation of a comparable particle arrangement.

Keywords: Powder mixture, effective permeability, mixing formulas, magnetostatic FE simulation.

Zusammenfassung: Dieser Artikel stellt einen neuartigen Simulationsansatz zur Bestimmung der magnetischen Eigenschaften von zweiphasigen und zufällig angeordneten Pulvermischungen vor. In der Industrie werden häufig eisenhaltige Pulvermischungen oder metallische Granulate als Grundmaterial für Herstellungsprozesse eingesetzt. Ihre Eigenschaften sind daher für den Materialhersteller von großem Interesse, um schließlich einen hohen Qualitätsstandard gewährleisten zu können. Typischerweise beeinflussen viele Parameter wie Zusammensetzung, Einschlussform und auch die Eigenschaften der Ausgangsstoffe selbst das makroskopische Verhalten derartiger Materialien. Insbesondere die resultierende Permeabilität von mehrphasigen und zufällig geordneten Materialien zeigt trotz konstanter Zusammensetzung eine starke Variation, welche auf die Partikelanordnung zurückgeführt werden kann. Für die Auslegung und Optimierung von Messaufbauten sind aus diesem Grund effiziente Simulatoren notwendig, um die effektive Permeabilität und ihren Schwankungsbereich abschätzen zu können. Zusätzlich zum Grundkonzept der neuen Simulationsmethode stellt dieser Artikel einige mögliche Auswertungen der simulierten Ergebnisse und deren Abhängigkeiten von den gewählten Simulationsparametern vor. Im letzten Jahrhundert haben sich für diesen Zweck in der Literatur viele verschiedene Mischformeln etabliert, die von den Autoren zusammengefasst und schlussendlich mit den Simulationsergebnissen verglichen werden. Am Ende wurden die simulierten magnetischen Eigenschaften mit jenen einer Finite-Elemente-Simulation verglichen.

Schlagwörter: Pulvermischungen, effektive Permeabilität, Mischformeln, magnetostatische FE-Simulation.

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1 Introduction

Metal foams, ferrous powder mixtures, metallic granulates, sintered materials, or multiphase solids are widely used in various production steps in industries. Especially during the steel making processes, it is of utmost importance to monitor the material quality to be able to influence the process chain immediately. Determination of a material's properties, for example compounding, in each state of fabrication is desired. The trend has been towards inline measurement techniques since several years [1]. The microstructure of all aforementioned materials is very complex, randomly ordered, and has a multi-phase nature. Hence, the properties of a composite can be strikingly different from the characteristics of its constituent materials [2].

Generally, the resulting electromagnetic behavior of products is influenced by the process as well as by the material composition. It is exactly this dependency that allows process or quality monitoring of material properties by determining their electromagnetic properties. In case of a steel making process, one of the most extensively used non-destructive techniques for inspecting material composition is measuring the electromagnetic parameters of a specimen by the eddy current testing (ECT) method [3].

In addition to the inline measuring system, precise models and simulators are necessary that allow an estimation of the material composition if permeability is known or vice versa. In many practical cases, it is not required to describe the magnetic field quantities in microscopic detail, i. e. the magnetization state of each element and the exact resulting flux density in each grain or powder particle is irrelevant. In this case, the concept of homogenization can be used to model the overall material property of a material under test [4]. Thereby, a complexly ordered material structure inside a defined volume or a periodically repeated arrangement within a specific volume is replaced by one block of homogeneous material that features the same size and an effective material parameter, as shown in Fig. 1. In case of compounds with different magnetic constituent properties, the observed effective mate-

Figure 1: Fundamental idea of homogenization applied on a randomly ordered mixture consisting of a gaseous background medium with μ_a and two magnetic phases with μ_1 and μ_2 .

Table 1: Summary of mixing models and their names in literature.

Sihvola's ν -formulas [7]		
	$\nu = 0$	Rayleigh [8] Maxwell-Garnett [10] Bohren-Batten [11] Wiener [12] Pearce [13]
$\frac{\mu_{\text{eff}} - \mu_1}{\mu_{\text{eff}} + 2\mu_1 + \nu(\mu_{\text{eff}} - \mu_1)} = f \cdot \frac{\mu_2 - \mu_1}{\mu_2 + 2\mu_1 + \nu(\mu_{\text{eff}} - \mu_1)}$	$\nu = 2$	Böttcher [14] Taylor [15] Polder-van Santen [16]
	$\nu = 3$	QCA [17] GKM [7] CPA [18]
Power law formulas [9]		
$\mu_{\text{eff}}^\beta = f \cdot \mu_2^\beta + (1-f) \cdot \mu_1^\beta$	$\beta = \frac{1}{2}$	Birchak [19]
	$\beta = \frac{1}{3}$	Looyenga [20]
	$\beta \rightarrow 0$	Lichtenecker [21]
$\frac{\mu_2 - \mu_{\text{eff}}}{\mu_2 - \mu_1} = (1-f) \cdot \left(\frac{\mu_{\text{eff}}}{\mu_1}\right)^{\frac{1}{3}}$	non sym.	Bruggeman [22]
Boundaries [9]		
Wiener [12]	Hashin-Shtrikman [23]	
$\mu_{\text{eff,max}} = f \cdot \mu_2 + (1-f) \cdot \mu_1$	$\mu_{\text{eff,max}} = \mu_1 + \frac{f}{\frac{1}{\mu_2 - \mu_1} + \frac{1-f}{2\mu_1}}$	
$\mu_{\text{eff,min}} = \frac{\mu_1 \cdot \mu_2}{f \cdot \mu_1 + (1-f) \cdot \mu_2}$	$\mu_{\text{eff,min}} = \mu_2 + \frac{1-f}{\frac{1}{\mu_1 - \mu_2} + \frac{f}{2\mu_2}}$	

rial parameter of the homogeneous specimen is called effective permeability μ_{eff} . When the effective material parameter is correctly defined and the specimen is magnetized, both the complex structure and its homogeneous equivalent would form the same external magnetic field distribution. According to [6], homogenization is a low frequency method and

$$d_{\text{max}} \ll \frac{\lambda_e}{2\pi} \quad (1)$$

has to be fulfilled, where d_{max} is the maximum size of the randomly ordered inclusions and λ_e is the wavelength of the external magnetization field. On the one hand, the resulting permeability μ_{eff} mainly depends on permeabilities of the two solid phases μ_1 and μ_2 as well as on the composition of the specimen defined by the volume fraction f of the solid phase 1. On the other hand, additional and usually unknown influences affect the effective material parameter such as the particle size, inclusion shape, temperature behavior, local packing densities, particle arrangement, magnetic nonlinearity, and so on. For this reason, many different models and mixing formulas with different requirements are applied in literature.

The authors collect both empirical rules determined by measurements and mixing formulas derived from electrostatic or magnetostatic theories and summarize them in Table 1 [5]. All mixing rules listed in the table are rewrit-

Figure 2: Three randomly ordered structures consisting of two magnetic phases (light and dark gray) and a non-magnetic background medium (white) with the same composition, but strongly different effective permeabilities.

ten for two-phase magnetic mixtures consisting of a solid inclusion phase with a permeability of μ_1 , a second solid inclusion phase with a permeability of μ_2 and a gaseous background medium (typically air) with a permeability μ_a . Sihvola et al. give an overview about two-phase mixing models in [6, 7, 8, 9] and merge some of them to one powerful explicit formula with a dimensionless parameter ν . If this modeling parameter ν , the magnetic properties of the constituents, and the effective permeability are known for a specific material under test, the composition $f(\mu_{\text{eff}}, \mu_1, \mu_2)$ can be easily calculated no matter for which formula. For some modeling approaches or computational optimizations of measuring setups the opposite and in case of Sihvola's ν -formula the very complicate relation $\mu_{\text{eff}}(f, \mu_1, \mu_2)$ is necessary. Because of this, simpler power-law formulas with a dimensionless modeling parameter β have been established in literature. Furthermore, Bruggeman [22] introduces a power law formula with a certain exponent one-third. Note that each model listed in Table 1 predicts another effective permeability for the same composition.

An additional challenge concerning the simulation as well as the measurement of the effective permeability is shown in Fig. 2. The illustrated specimens show different possibilities to arrange the same number of particles on an ideal squared grid. Despite of the same composition f , any arrangement evidently exhibits another permeability at a given magnetization direction. Thus, a variation of the effective permeability μ_{eff} must be expected because of the random particle distribution inside the volume. In order to specify the accuracy of a new developed and optimized inline measuring setup, the permeability fluctuation due to the particle distribution and its possible dependency on the composition f must be taken into account. Hence, in Table 1, boundaries are defined that represent theoretical limits for the effective permeability and allow an estimation of the possible fluctuation range.

Figure 3: Comparison of the effective permeability based on well-known mixing formulas and bounds from Table 1 using the parameter set $\mu_1 = 1$, $\mu_2 = 150$ and $f = 0 \dots 1$.

Figure 3 compares the courses of the presented mixing formulas and the theoretical limits for a certain parameter set. However, it cannot be determined in advance, which mixing formula and boundary describe the magnetic behavior of a particular material best. For this reason, fast simulators are necessary in order to generate many different particle arrangements and to determine their effective permeability and its variation depending on the simulation parameters.

2 Novel and efficient simulators

Because of all the aforementioned aspects, different authors tried to implement simulators that are able to simulate the effective magnetic and electric behavior of powder mixtures, materials microstructure, or idealized microscopic arrangements. Therefore, different concepts and methods were established. As first approach, optical photographs of microstructures are made to obtain realistic geometries of the microstructure [24, 25, 26, 27]. After a conversion in black and white pictures, this geometries are imported into commercial simulation programs for the numerical calculation of electromagnetic fields to estimate their resulting effective permeability. However, creating and editing images of textures, as well as the numerical determination of the effective material parameters of these complex geometries is very time-consuming. Especially the simulation of a huge amount of arrangements at various volume fractions and diverse raw materials is very inefficient in this manner. For each variant a particu-

lar material has to be produced. Hence, in [28, 29, 30, 31] the authors used idealized geometries for modeling purposes instead of real textures. In that case, elements of the solid inclusion phase are randomly or orderly placed inside a background medium in form of ideal three- or two-dimensional geometric objects such as spheres, cubes, cylinders, needles, ellipsoids, circles, squares and so on. On the one hand, the generation of complex but idealized microstructures is fast and efficient. But on the other hand, all aforementioned authors use numeric simulation tools such as the finite element method (FEM) to calculate the resulting effective parameters of the very complex geometries. Thus, the method inherently stays slow. In [32] the authors aimed to establish a simple method to evaluate the permeability of sintered materials without using real texture images and numerical solving techniques. Accordingly, a magnetic circuit method that represents the texture by a resistor network has been employed. The novel simulator presented in this paper was developed based on this idea.

2.1 Basic principle of the novel simulator

The novel powder simulator is based on the assumption that fine and complex structures can be decomposed into cubes (3D) or squares (2D). In this paper, the random assignment of the permeability is limited to a plane and along the third dimension magnetic properties are constant. In principle, the introduced method allows an extension to the third dimension, but this was not done to present the basic concept here. Figure 4a shows a homogeneously magnetized block consisting of $N = n \cdot m$ particles, where m and n are the number of elements in horizontal and vertical direction, respectively.

In the beginning, all N elements are initialized with the physical properties of the gaseous phase that fills up the resulting pores in the final material in the end of the procedure. Next, the two solid phases are placed on coordinates calculated by a random number generator. Generally, its probability density function affects the spatial distribution of the various phases within the sample volume and consequently influences the resulting magnetic properties of the total specimen. In order to ensure different and realistic powder mixtures, uniformly distributed random numbers are used for the positioning of the different material phases. On the one hand, the total number of inserted inclusions of solid phase 1 is defined by $N_1 = N \cdot (1 - f) \cdot \lambda$, where f is the volume fraction of the inclusion phase and $\lambda = \frac{\rho_{\text{pressing}}}{\rho_{\text{solid}}}$ defines the relative density of the pressed compound [33]. On the other hand,

Figure 4: (a) An idealized model of a randomly ordered mixture consisting of nine discrete particles, two solid phases with μ_1 and μ_2 , and one gaseous phase with μ_a . (b) The equivalent electrical circuit of the model for the permeability analysis with SPICE.

$N_2 = f \cdot \lambda \cdot N$ elements corresponding to the second solid phase with μ_2 are considered. In practice, it is not possible to reach solid body density by pressing fine particles with finite energy. Therefore, the final mixture typically includes trapped air and reaches relative densities $\lambda < 1$. This additional gaseous phase is represented by the remaining elements $N_a = (1 - \lambda) \cdot N$ and exhibits the permeability of air $\mu_a \approx 1$. The magnetic properties can be randomly assigned to the elements at a ratio of N_1 , N_2 , and N_a . This approach is shown in this paper. In contrast, it would be possible to create ordered structures (like crystals) or randomly placed clusters of several particles and different material characteristics, such as particles with coating or the microstructure of multi-phase steels.

After allocating the defined physical properties to all elements, the structure in Fig. 4a is converted into an equivalent electrical network shown in Fig. 4b. Thereby, each element is represented by an equivalent resistor network including four star-connected resistors. In general, their resistances depend on the geometric dimensions a and b , the absolute permeability μ_0 , and the relative permeabilities μ_1 and μ_2 of the materials. In the case of cubic particles ($a = b$) the values of resistance are defined as

$$R_1 = \frac{1}{2a\mu_0\mu_1} \quad (2)$$

for particles of the solid inclusion phase 1,

$$R_2 = \frac{1}{2a\mu_0\mu_2} \quad (3)$$

for elements of the solid inclusion phase 2 and

$$R_a = \frac{1}{2a\mu_0} \quad (4)$$

for the remaining regions filled with air, where a is the side length of one square element shown in Fig. 4a and μ_0

the absolute permeability of vacuum. In principle, the star-shaped subnetwork used in this paper is only one possibility to model the magnetic property of one element, but it is the simplest network permitting a magnetic flux distribution in both spatial directions. The possibility for investigating different equivalent networks is an additional advantage of the introduced simulation concept. For example, the resistor network of a single particle can be easily extended by additional transition resistances to model synthetic coatings or oxide layers for each particle with other physical properties than the residual material. Additionally, various network structures can be assigned to each element in order to model different material properties. Furthermore, concerning the generation of the structure and the equivalent network, one is not limited to just two solid phases, as discussed here. Theoretically, each element can exhibit its own resistance values and, therefore, its individual magnetic characteristics. It is even possible to increase or decrease one of the four resistances inside one particle to obtain a preferential magnetic direction. Overall, the introduced concept offers great flexibility in modeling.

The magnetization of the specimen by an external field coil can be described by a voltage source with an impressed voltage V_{mag} . It generates a current I_{mag} that represents the resulting average magnetic flux through the specimen. Finally, the effective permeability can be determined by

$$\mu_{\text{eff}} = \frac{I_{\text{mag}}}{V_{\text{mag}}} \frac{m}{na\mu_0} \quad (5)$$

using the equivalent resistance of the network and the geometric parameters of the specimen. The authors use the open source electronic circuit simulator SPICE [34] to calculate I_{mag} . Of course, SPICE can handle nonlinear resistor networks as well. For this reason, saturation effects can be modeled by nonlinear relationships for the resistors $R_i = f(\mu_i)$ and thus the whole concept is not limited to the linear case, only.

2.2 Simulation results

In order to compare simulation results with well-known mixing formulas and bounds from literature the relative density λ must be set to 1 in all simulations, because all models described in Table 1 are only valid for that case. In Chapter 2.1 it was already shown that the novel simulation approach has a very large number of free parameters to guarantee maximum flexibility. Thus, only a small overview of possible simulation studies can be presented

Figure 5: Comparison of the normalized effective permeabilities based on well-known mixing formulas or bounds with 1,000 different simulated permeabilities using the simulation parameters: $f = 0 \dots 1$, $\lambda = 1$, $m = n = 150$, $\mu_1 = 1$ and $\mu_2 = 150$.

in this paper. To evaluate the qualitative course of the effective permeability independently of the selected phase permeabilities, the authors introduce a normalized effective permeability

$$\mu_{\text{eff},n}(f) = \frac{\mu_{\text{eff}}(f) - \mu_1}{\mu_2 - \mu_1}. \quad (6)$$

Figure 5 demonstrates 1,000 simulated effective permeabilities at different volume fractions f with a specific parameter set defined in the caption and compares them with the aforementioned mixing formulas and bounds in Table 1. Each individual simulation result is represented as a single point in the diagram and all simulated points together form the point cloud shown in Fig. 5. Notice that the total set of simulated data is clearly within the theoretical limits, but fluctuates significantly less than predicted and theoretically allowed by Wiener and Hashin-Shtrikman bounds and thus confirms the results in [9]. The qualitative course of the normalized effective permeability actually differs in the entire range of f from the well-known mixing formulas from the literature. Consequently, none of the known rules is suitable for describing this special kind of randomly arranged and squared particles. Differences in the prediction of $\mu_{\text{eff}}(f)$ by theoretical mixing rules, measurements and simulations are also shown in [24], [25] and [31]. On the one hand, this difference occurs because of the necessary assumptions and simplifications made when deriving the mixing laws. Therefore, they only apply to mixtures with special and restricted properties. On the other hand, the simulators contain assumptions and simplifications, whereby the resulting courses

Figure 6: Normalized average effective permeability $\bar{\mu}_{\text{eff}}$ calculated for one specific parameter set $f = 0 \dots 1$, $\lambda = 1$, $\mu_1 = 1$, $\mu_2 = 150$ and different number of particles m, n .

Figure 8: Normalized average effective permeability $\bar{\mu}_{\text{eff}}$ calculated for one specific parameter set $f = 0 \dots 1$, $\lambda = 1$, $m = n = 150$ and different permeabilities μ_2 .

Figure 7: Resulting range of fluctuation of the simulated effective permeability related to the average permeability depending on the number of particles m, n using constant simulation parameters: $f = 0 \dots 1$, $\lambda = 1$, $\mu_0 = 1$, $\mu_1 = 150$.

Figure 9: Resulting range of fluctuation of the simulated effective permeability related to the average permeability depending on the permeability of inclusion phase μ_2 using constant simulation parameters: $f = 0 \dots 1$, $\lambda = 1$, $m = n = 150$, $\mu_0 = 1$.

contain a modeling error as against real structures as well. Especially for small volume fractions $f \leq 0.6$ the mixing formula according to Bruggeman describes the simulated data best.

The comparison with the other formulas in Fig. 5 contains one thousand simulations with only one parameter set. However, the great strength of the new method is the efficient and fast simulation of many different particle structures with various parameters in order to determine their effects on the averaged course $\bar{\mu}_{\text{eff}}(f)$ as well as on the fluctuation of the effective permeability. In the

following the standard deviation $\sigma(f) = \sqrt{v(f)}$ is used to quantify the amount of fluctuations of the simulation results, where v is the estimated variance of the data related to the normalized average effective permeabilities $\bar{\mu}_{\text{eff}}(f)$. Figures 6 and 7 show the effect of the total particle number defined by m and n on the course and variation range of the effective permeability $\mu_{\text{eff}}(f)$. Due to the constant volume of the specimen, in the following a higher number of elements implies smaller particles and hence more finely ground powders. Note that the number of particles has no significant effect on the mean permeability behavior.

ior in Fig. 6. All curves converge with an increasing refinement toward the average permeability course with $m = n = 150$. Hence, the maximum permeability difference between 10,000 ($m = n = 100$) and 22,500 ($m = n = 150$) elements amounts to less than 1%. The effect on the fluctuation of the effective permeability is significantly greater. According to Fig. 7, all simulated powder mixtures exhibit no considerable deviation of the effective permeability at volume fractions close to 0 and 1. Especially coarse mixtures, i. e. both simulation parameters m and n take small values, feature rather strong variations at $0.5 \leq f \leq 0.7$ up to nearly 150 %. This can be explained by the fact that one phase clearly predominates at these volume fractions and that therefore the exact position in the volume of the minor phase is irrelevant. In contrast, at volume fractions around $f = 0.6$, both solid phases (when $\lambda = 1$) are almost in balance and thus paths consisting of the magnetically more conductive phase are formed or not. If a closed path is developed and no magnetic saturation occurs during the route, the majority of the total magnetic flux can close with low resistance through the sample and the effective permeability increases abruptly. The volume fraction f_c at which this transition (closed path or not) takes place is called percolation threshold and is predicted by percolation theory for an ideal 2D square lattice with site-percolation as $f_c \approx 0.59$ [35]. Consequently, our simulation results can fulfill the theoretical forecast by the percolation theory quite well.

In contrast to the total number of particles N , the permeability of the solid phases affects the behavior of $\bar{\mu}_{\text{eff},n}$ as shown in Fig. 8. Generally, the smaller the permeability of the magnetically more conductive phase μ_2 , the less flux is linked in the magnetic particles and tied in the magnetic path through the sample, respectively. Especially at very high values of μ_2 , the resulting permeability μ_{eff} remains small compared to μ_2 as long as no or only a few magnetic paths can be formed through the specimen ($f < f_c$). If the percolation threshold f_c is exceeded, μ_{eff} increases rapidly until the end value $\mu_{\text{eff}} = \mu_2$ is reached at $f = 1$. When, in contrast, the magnetically conductive phase has a low permeability, the discussed transition is weaker, because closed paths do not have such a large effect on the total magnetic flux through the specimen. The dependency of μ_2 also converges and a permeability of $\mu_2 > 600$ is sufficient to keep changes of the normalized curves induced by μ_2 lower than 1%. In fact, the deviation of the permeability due to the change of permeability μ_2 in Fig. 9 has its maximum around the theoretically predicted percolation threshold $f_c \approx 0.59$ as well. Overall, Figs. 5–9 include 11,000 simulated effective permeabilities using various particle arrangements with 11 different parameter sets.

Figure 10: Distribution of the magnetic flux density $|B|$ by FEMM [36] over the total computing area with the parameter set $\lambda = 1$, $f = 0.5$, $m = n = 10$, $\mu_1 = 1$ (light gray), $\mu_2 = 150$ (dark gray) and an estimated effective permeability of $\mu_{\text{eff}} = 7.16$.

Figure 11: Distribution of the magnetic flux density $|B|$ by FEMM [36] over the total computing area with the parameter set $\lambda = 1$, $f = 0.5$, $m = n = 10$, $\mu_1 = 1$ (light gray), $\mu_2 = 150$ (dark gray) and an estimated effective permeability of $\mu_{\text{eff}} = 10.35$.

3 Comparison with FEM

The introduced simulation method fulfills the theoretical predictions of percolation theory as mentioned before. Nevertheless, the calculated curves for the effective permeability and its deviation have to be additionally checked and compared with the results of a numeric tool. For this reason, the authors use the freely available FE software FEMM [36]. Figures 10 and 11 show two representative examples of randomly ordered two-phase arrangements consisting of 100 elements ($m = n = 10$). In the lower left

corner the ideal particle distribution is shown. The resulting magnetic flux density B as well as the magnetizing force H inside the total computing area are calculated by solving the magnetostatic problem with FEMM. In order to expose the sample to a homogeneous magnetic field, the boundary conditions (BC) can be defined as shown in Fig. 10. Thereby the vector potential A along the upper and lower side edge of the computing region is defined as constant values A_h and A_l , respectively. Additionally the undefined boundary condition of FEMM is used to force perpendicular flux lines at the left and right boundary. Hence, the specimen is magnetized by an homogeneous and constant magnetic field over the whole computing region. All following simulations were performed with $A_h = 40$ mVs/m and $A_l = 0$ Vs/m. Undefined boundary conditions in FEMM imply that the magnetic flux density is normal to the boundary. Finally, the effective permeability of the mixture can be determined by

$$\mu_{\text{eff}} = \frac{B_{\text{avg}}}{\mu_0 H_{\text{avg}}}, \quad (7)$$

where B_{avg} is the average magnetic flux density inside the sample, μ_0 the absolute permeability in vacuum and H_{avg} the average magnetization force inside the specimen. The simulated samples in Figs. 10 and 11 exhibit a volume fraction of $f = 0.5$, but differ in their calculated effective permeability μ_{eff} . This deviation is a result of the different particle arrangements. In contrast to the sample in Fig. 10, the specimen in Fig. 11 percolates already, for which reason more magnetic flux is linked in a completely closed path. The resulting effective permeability is higher.

Both, the average of the normalized effective permeability $\mu_{\text{eff},n}$ computed by FEMM as well as its fluctuation range for different particle sizes are compared with the aforementioned curves in Fig. 12 and 13. Results with the shortcut “RN” (= resistor network) are calculated with the novel simulation method. Especially if one solid phase dominates the magnetic behavior ($f \approx 0$ or $f \approx 1$), the estimated values of permeability and its deviation fit very well. For larger particle numbers N , the difference is almost negligible. The divergence in the range around the percolation threshold f_c are caused by the different possibilities of flux linkage through the specimen permitted by the different simulation concepts. While the network method only allows closed magnetic paths over the side edges of the squares, the magnetic flux in the field problem can certainly close over the corners as indicated by Fig. 11. As a result, magnetically preferred paths can be formed easier in the field problem and the effective permeability tends to greater values when both phases are in

Figure 12: Normalized average effective permeability $\bar{\mu}_{\text{eff}}$ calculated by FEMM [36] for the same parameter set as in Fig. 6.

Figure 13: Fluctuation range of the effective permeability calculated by FEMM [36] for the same parameter set as in Fig. 7.

balance. On the one hand, this problem could be solved by choosing a resistor network that allows a flux linkage through the element corner. On the other hand, transitions between particles via ideal corners cannot occur in practice, for which reason they should be avoided in the simulation. Another possibility could be a replacement of the ideal squares by more suitable geometric shapes, such as hexagons. To compare, the simulation results determined by numeric field calculation can confirm those of the novel simulation method presented in this paper.

Finally, a comparison of the required computing time of both methods in Table 2 shows once again the great advantage of the novel method. Both presented concepts

Table 2: Comparison of the total computing time for one arrangement of the novel concept (RN) $\bar{t}_{\text{RN}}$ and FEMM [36] $\bar{t}_{\text{FEMM}}$.

Number of elements N		$\frac{\bar{t}_{\text{FEMM}}}{\bar{t}_{\text{RN}}}$
100	($m = n = 10$)	4.0
625	($m = n = 25$)	5.2
2,500	($m = n = 50$)	6.1
10,000	($m = n = 100$)	6.8
22,500	($m = n = 150$)	9.2

transfer the magnetostatic problem to a linear system of equations. Generally, solving the FE-problem is faster than the calculation of I_{mag} by SPICE. However, the creation and modification of the geometry and the material assignment, as well as the correct meshing of the computing area is more time-consuming, such that the resistance method is more efficient in the end. According to Table 2, the finer the rectangular or squared lattice, i. e. the greater the number of elements N , the more efficient is the novel method presented in this paper compared to FEMM.

4 Conclusion

In this paper, a novel and efficient simulator for randomly ordered structures consisting of two or more phases is introduced and compared with a numerical field calculation tool based on finite elements. The main advantage of the novel simulation method compared with existing techniques is its high flexibility relating to the generation of complex, randomly, or orderly arranged microstructures combined with a low computing time. For example, more than 10,000 different particle arrangements were simulated and evaluated in a short time.

Overall, the simulation results of both the novel approach and the FE-based concept fit together very well and fulfill predictions made by established theories such as the percolation theory. Additionally, the novel simulation concept can be easily extended to powders that are more complex and sintered materials used in industry.

In a future work, the forecast of permeability variations due to parameter changes will be studied. Especially the nonlinear relationship between B and H of ferro- and ferrimagnetic materials can strongly influence the behavior of the effective permeability $\mu_{\text{eff}}(f)$. For this reason, the simulator will be extended to nonlinear material properties in order to compare the simulation results with measurements of bulk materials consisting of magnetic and non-magnetic constituents.

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