Application of variable excitation frequency for amplitude reduction

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Dynamic systems are faced with large vibration amplitudes if they operate near the resonance frequency of the system or while passing through the resonance. With a suitable external excitation a reduction of the amplitude of the oscillation can be achieved. The analysis of a variable excitation frequency was first carried out on the force driven single degree of freedom harmonic oscillator. The influence of various excitation parameters can be seen after a mathematical transformation of the excitation signal. The elimination of the natural frequency of the system from the excitation signal is possible with specially selected parameters. It is demonstrated that suitable numerical values, correlations between the parameters and functions allow to reduce the oscillation of the amplitude of the system if the operation takes place near the resonance frequency and also during the resonance passage. The theoretical assertions are validated with an experimental setup and a parameter identification has been made. In addition a worst case scenario is analysed on the test rig. The developed requirements are extended to force driven harmonic oscillators with multi degree of freedom.


1 Introduction

Based on a linear oscillator, one mass connected to the ground by a Kelvin-Voigt-Element, specific excitation force functions for the reduction of the vibration amplitude have been presented by the authors in [1]. A defined modulation of the frequency has been added to the excitation frequency or to the linearly increasing excitation frequency near the resonance frequency.

2 Experimental Setup

In order to evaluate the theoretical statements, we realized an experiment of a linear oscillator, see Fig. 1a. For that experiment we took a beam with a mass $m = 1$ kg at the tip and the force excitation is acting at the system with a shaker near the clamping of the beam. There is one force sensor attached to the beam and one acceleration sensor attached to the mass. The spring stiffness is calculated with the resonance frequency of the system: $c = (2 \cdot \pi \cdot 9.125 \text{ Hz})^2 m = 3287.2 \text{ N/m}$. The damping factor $d = 0.14 \text{ kg/s}$ is determined from the decaying of the signal after a short impact and for the simulation the force is transformed to the position of the mass. The corresponding simulation model is presented in [1].

![Fig. 1: a) Structure of the experiment, b) Acceleration signal at resonance frequency or modulated excitation frequency, c) Acceleration signal with linearly increasing excitation frequency and additional modulation](image)

The acceleration signals of the performed experiments for an operation near the resonance frequency are depicted in Fig. 1b. With the mentioned modulation and the right choice of the parameters the acceleration is not increasing like a system in resonance. The maximum amplitude of the small signal is $0.766 \text{ m/s}^2$ in the simulation and $0.787 \text{ m/s}^2$ in the experiment.

Fig. 1c shows the run-up through the resonance frequency. In the case of the targeted modulation, the amplitude of the oscillation remains much smaller than with the linearly increasing excitation frequency. The ratio of the maximum amplitudes is $0.96/3.1 = 0.31$ in the simulation and $1.1/3.05 = 0.36$ in the experiment. It is seen that the maximum amplitude can be reduced by more than 60 percent compared to a linearly increasing excitation frequency.

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If the amplitude spectrum of the force excitation signal in Fig. 2a is considered, it can be seen that there is a maximum near the minimum of the amplitude. In the worst case, the resonance frequency is missed. It follows that the maximum force amplitude occurs at the natural frequency of the system and the oscillation amplitude increases. In our worst test case, see Fig. 2b, the wrong chosen frequency is 8.605 Hz instead of 9.125 Hz. The ratio of the maximum acceleration amplitudes from the experiment is 1.27. The result of the simulations and the experiments is that the vibration amplitude can be reduced by more than 60 percent in the best case, or increased by about 30 percent in the worst case.

### 3 Application to a Multi Degree of Freedom System

The derived force excitation functions can also be applied to a multi degree of freedom system with \( n \) coordinates. For a system of linear differential equations the eigenfrequencies of the undamped system can be calculated with the homogeneous differential equation.

\[
M \ddot{x} + D \dot{x} + K x = F \quad M \ddot{x} + K x = 0 \quad \det (K - \omega^2 M) = 0
\]  

For each eigenfrequency of the system, that appears in the force excitation function the modulation frequency \( \nu \) needs to be calculated. Considering the roots of the Bessel function \( J_0 \), see [2], and the amplitude \( \hat{\omega}_i \) of the modulation the excitation force is

\[
F_n(t) = \hat{F} \sum_{i=1}^{n} \sin (\omega_0 t - \eta \cos (\nu t)) \quad \text{with} \quad \nu_i = \frac{\hat{\omega}_i}{\eta_{\nu_i=0}} \quad \text{and} \quad \hat{\omega}_i = p \omega_0. 
\]

For the passage through the resonance \( n \) is equal to the number of natural frequencies and the assumptions from [1] have to be considered for each eigenfrequency.

\[
F_n(t) = \hat{F} \sin \varphi(t) \quad \text{with} \quad \varphi(t) = \int \left( \alpha t + \sum_{i=1}^{n} \omega_i \sin (\nu (t - t_{i1}) - \pi) \right) \, dt \quad \text{and} \quad \varphi_0 = 0
\]

The amplitude spectrum of the force excitation function of a two degree of freedom system with arbitrarily chosen numerical values is shown in Fig. 2c. The two eigenfrequencies of the system are eliminated from the excitation force signal. The distance between the sideband frequencies is bigger at higher frequencies due to the definition of the amplitude of the modulation, with 10 percent of the corresponding resonance frequency. In the case of a resonance passage with frequency modulation, the amplitudes of the force excitation signal are small at both natural frequencies of the system. The linearly increasing excitation frequency is modified at two different time periods. The amplitude of the deflection of the masses is smaller in the cases of appropriate modulated frequency.

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**References**
